Analysis of plate with large elliptical hole and two smaller circular holes

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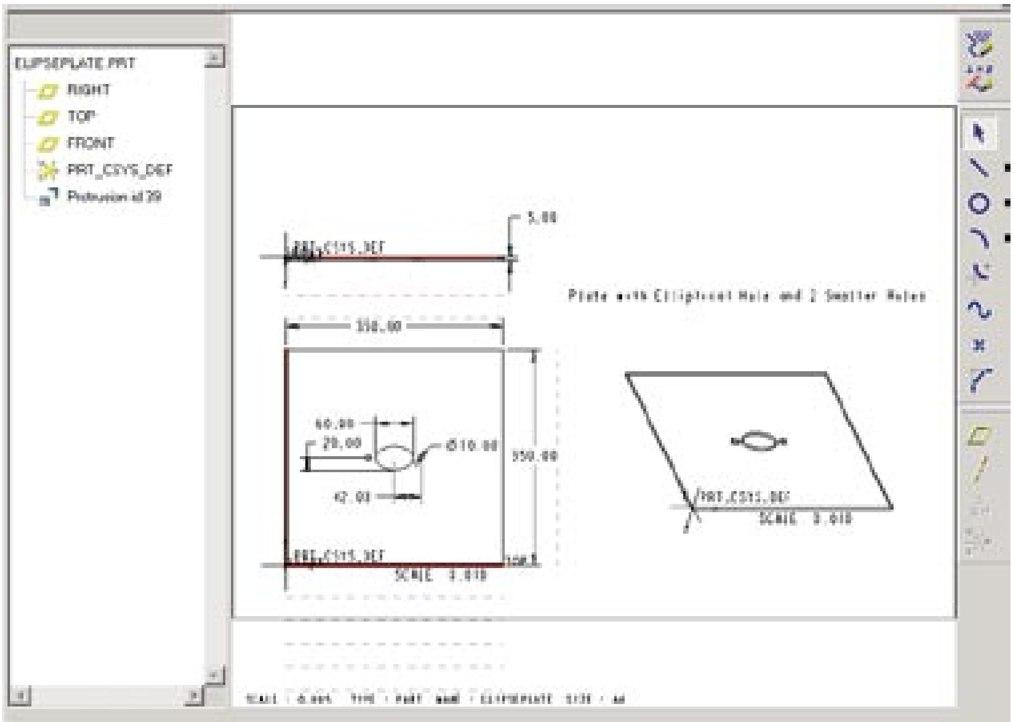
April, 2003

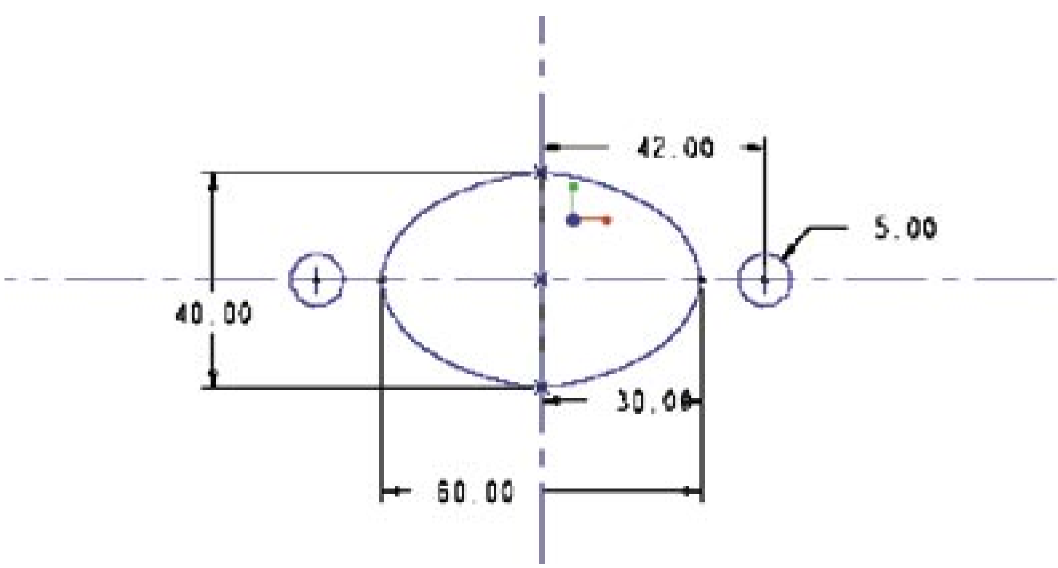
I hereby declare that this is my original work and research.
signed:

Statement of the Problem

The following plate, modeled in ProEngineer, is analyzed using Patran/Nastran finite element modeling software. Material is given as Aluminum with a yield stress of 300MPa.



Based on a student number that ends with 1, the distance required for modeling between the center of the ellipse and the center of the smaller circular hole is 40 +2*(last digit) = 42mm.



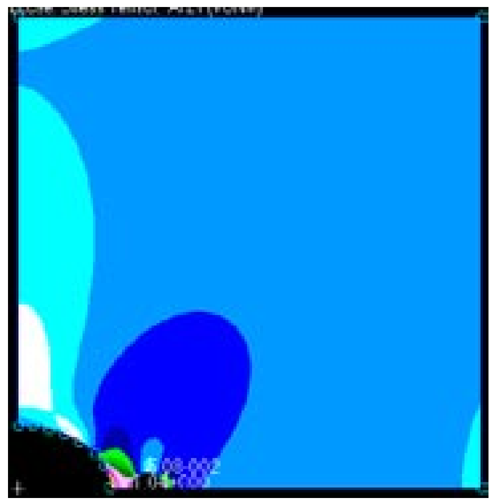
Since the plate is symmetrical in both the x and y directions, a quarter of the plate is modeled using 2D finite element analysis with a plate thickness of 5mm.

Question 1. Determination of the length of the plate

Here the objective is to find a length of the plate for finite element modeling so that the chosen length does not interfere with the desired results. It is important to choose a length so that the stresses on the edges are independent on the actual distribution of stresses within the plate. General rules of thumb indicate that nominal stresses become independent of localized stresses at roughly 4 times the region of the localized stress. I experimented with four different sizes for the length and width of the plate, ranging from roughly three to five times the distance between the elliptical hole and the circular hole. The results are shown below:

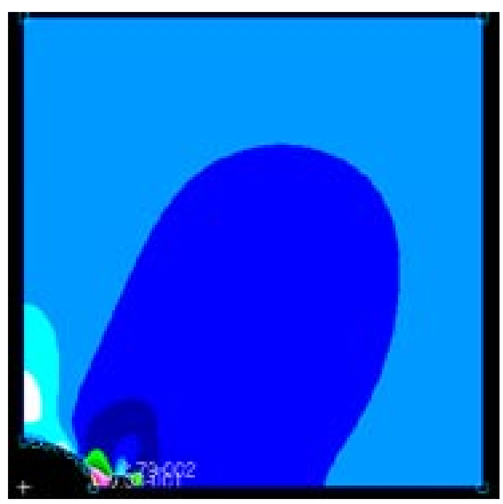
Initial Length Determination FEA models

Length =125mm



Length =150mm

Length =175mm



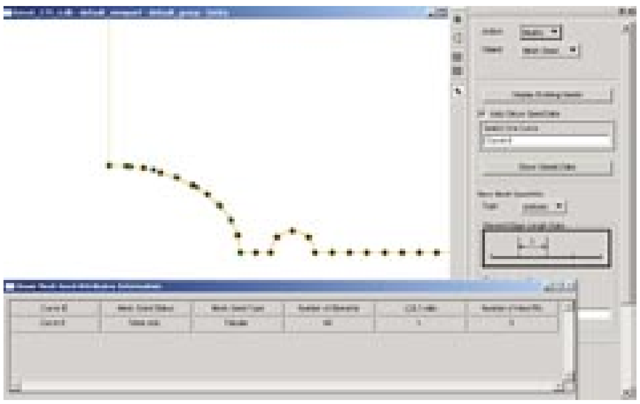
Length = 200mm

As can be seen from the Patran screen shots above, with lengths of 125mm and 150mm, the stress distribution on the boundary varies along the top and right edges. Since we are looking for a length that does not affect the results, a longer length is required. The 200mm model satisfies this condition, but because it is larger than necessary, excessive computational time would be required. Therefore, the 175mm length is chosen as it satisfies the requirement that the stress distribution along the boundaries is constant.

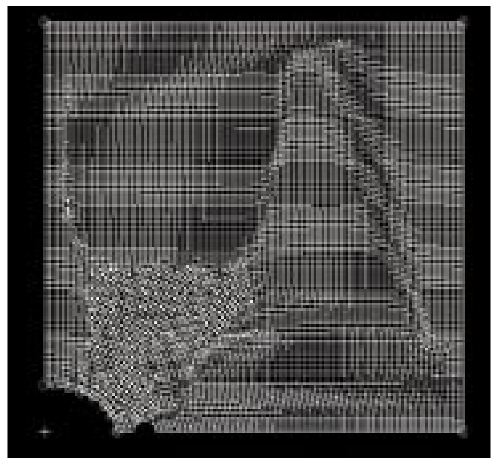
Question 2: Determination of Compound Stress Concentration

Creating the mesh

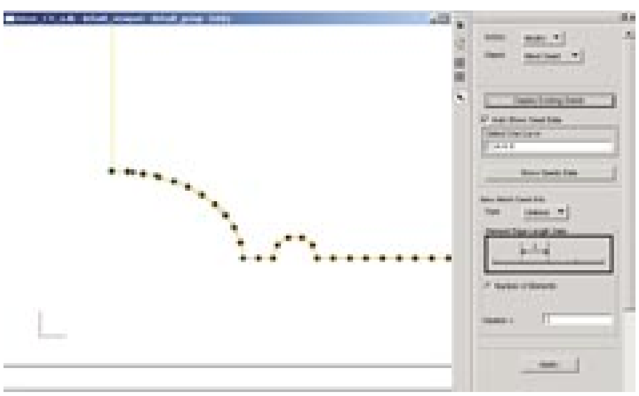
With a model edge length of 175mm, the next step was to determine the mesh characteristics. Various meshes were created utilizing a 2D model. Isomeshes solved adequately, yet the results varied considerably. The Paver method was chosen for its more consistent meshing around the tight curves in the area between the two holes. Since the intent is to create a sequence of three grids in order to calculate convergence, each with relative mesh sizes in the ration of 1:1/2:1/4, it is important to be consistent with the input characteristics such as the mesh seeding. The bottom curve was chained together and seeded. Experiments and chosen methods are described below.



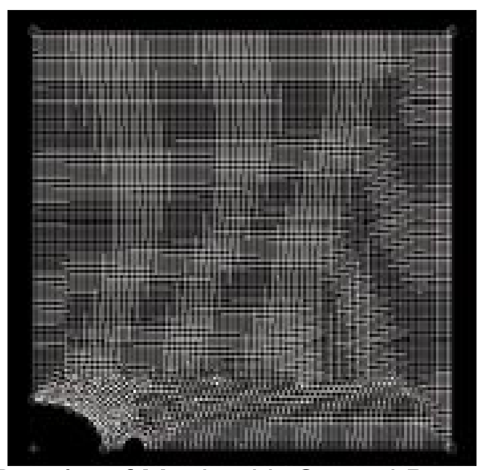
Uniform Element Length mesh seed.
Element Length = 4mm.



Results of Mesh with Uniform mesh seed.



Curve Based mesh seed for 4mm mesh.
Element Constraints: Min Length = 3mm
Max Length = 4.8 mm



Results of Mesh with Curved Based mesh seed (all other parameters the same as above).

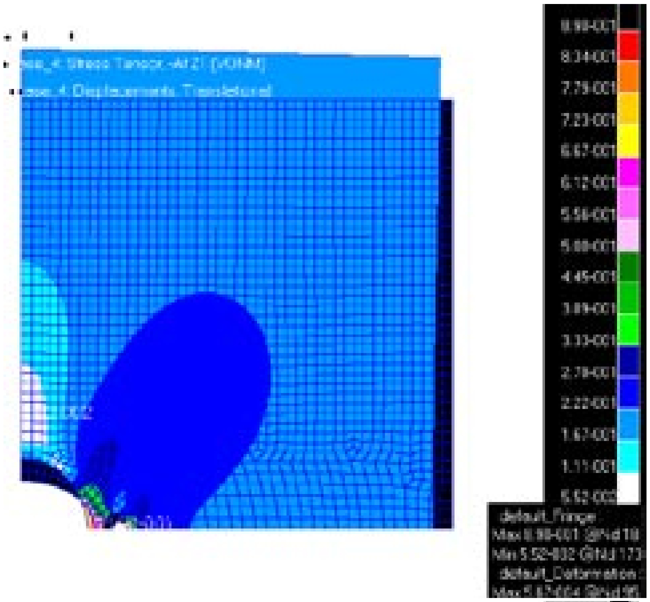
As can be seen above, using the Curve Based mesh seed resulted in a more consistent mesh. In order to preserve consistency, the following constraints are used with a Curved Based seed:

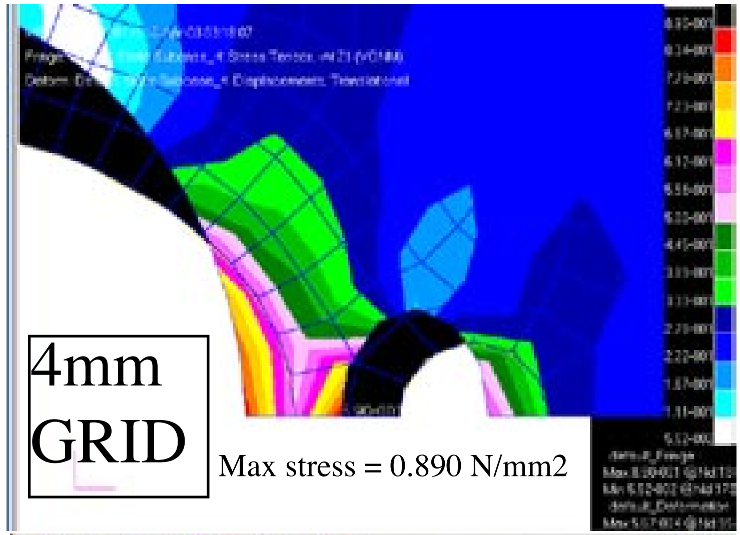
Minimum Length = 0.75 * Mesh element size

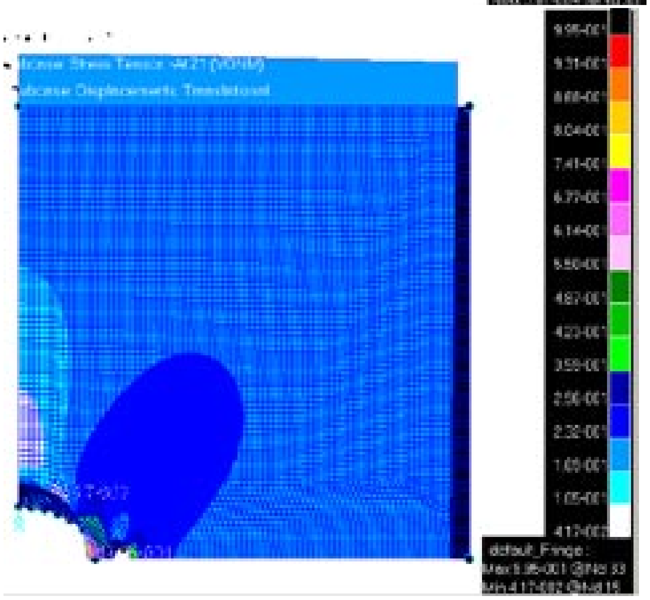
Maximum Length = 1.2 * Mesh element size

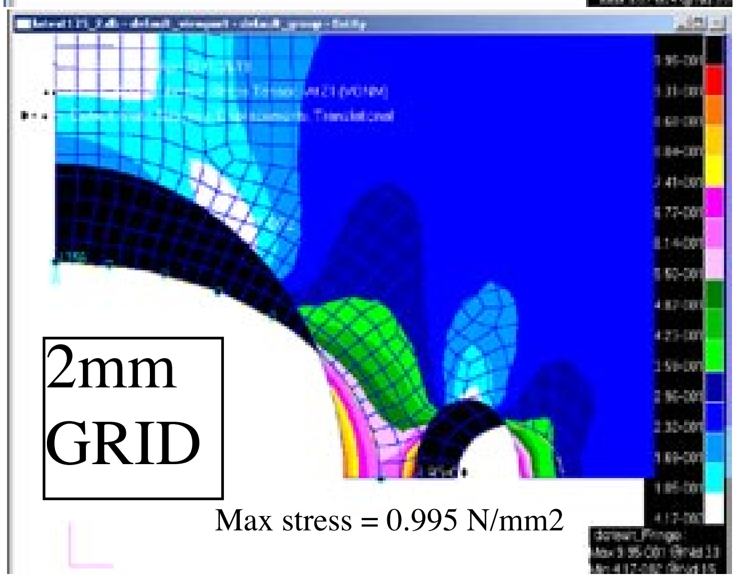
Three models were then created using Paver and Quad 4 elements (since no bending moments are present), using a mesh seed on the lower curve as described above, with mesh element sizes of 4mm, 2mm, and 1mm. Symmetry boundary conditions are applied to the left and bottom edges, and the node at the top center of the elliptical hole was fixed. The material is Aluminum with an elastic modulus of 71000 N/mm2 and a Poisson ratio of 0.334. A distributed load is applied to the top elements of 1 Newton/mm.

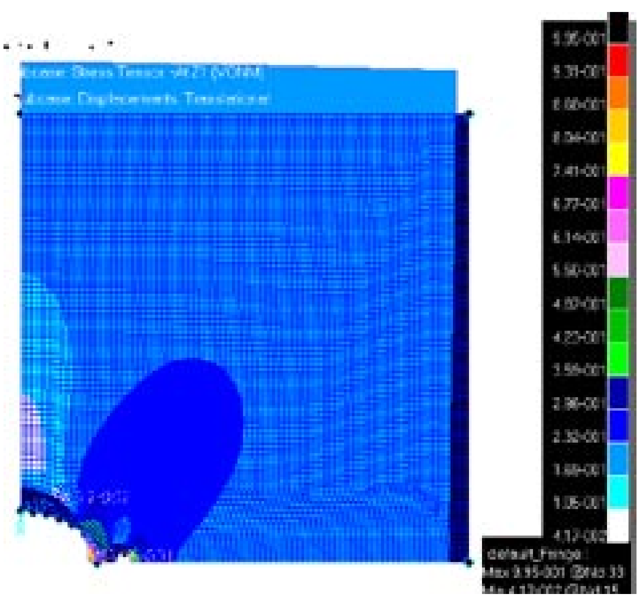
The FEA Results:

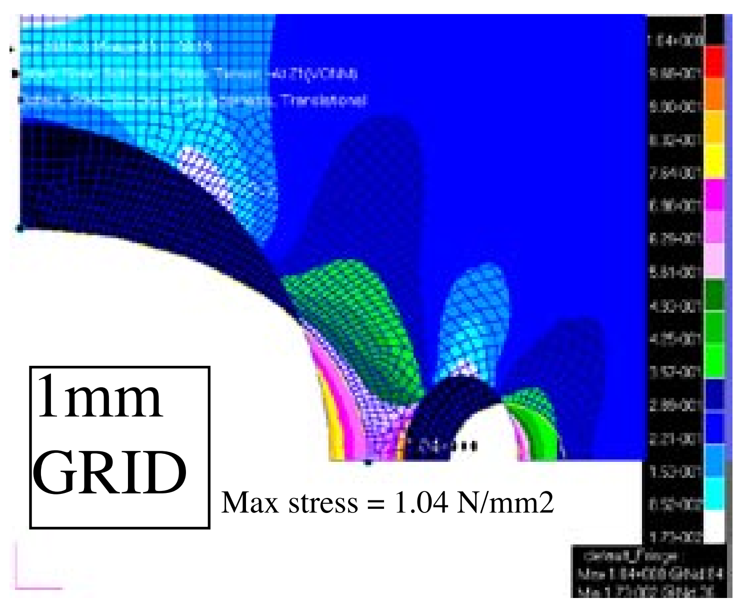








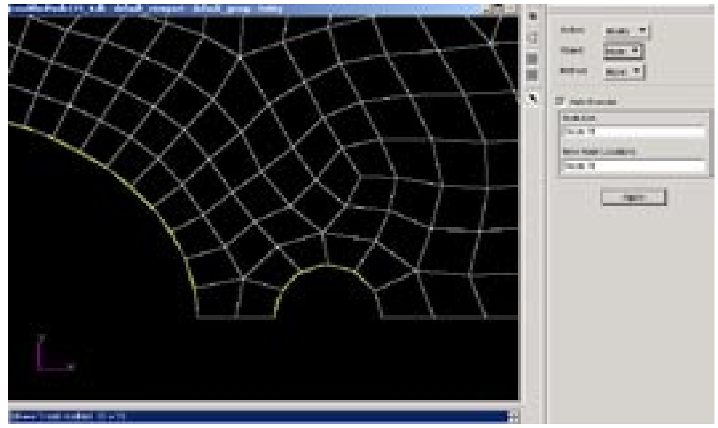




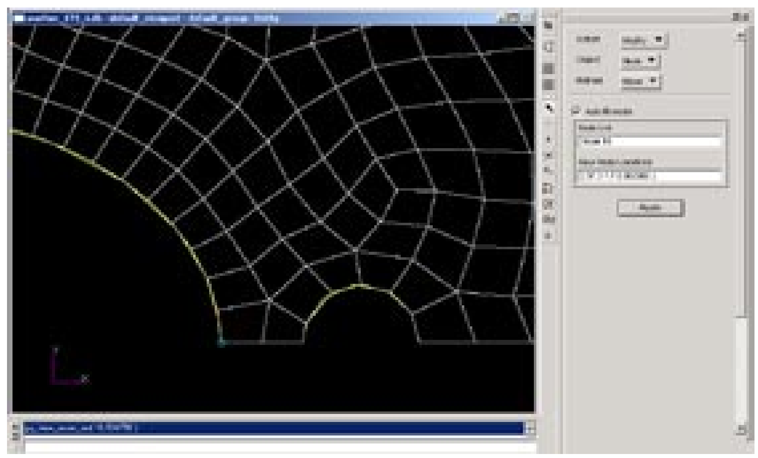
Results continued

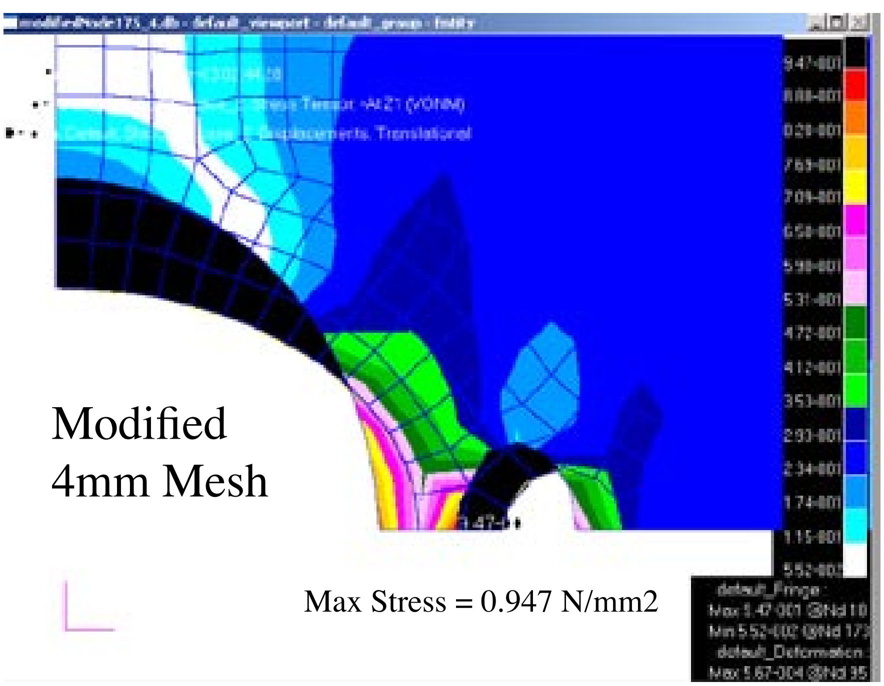
Prior to discussing the results, it is worth looking at the effect of the mesh on the numerical results of the FEA computations. In the case of the 4mm grid, the precise placement of the nodes is critical to the results. An example is illustrated below:

Mesh prior to
Modify->Node -> Move



Mesh after
Modify->Node -> Move
(a single node on the left side of the circular hole moved closer to the location of maximum stress).





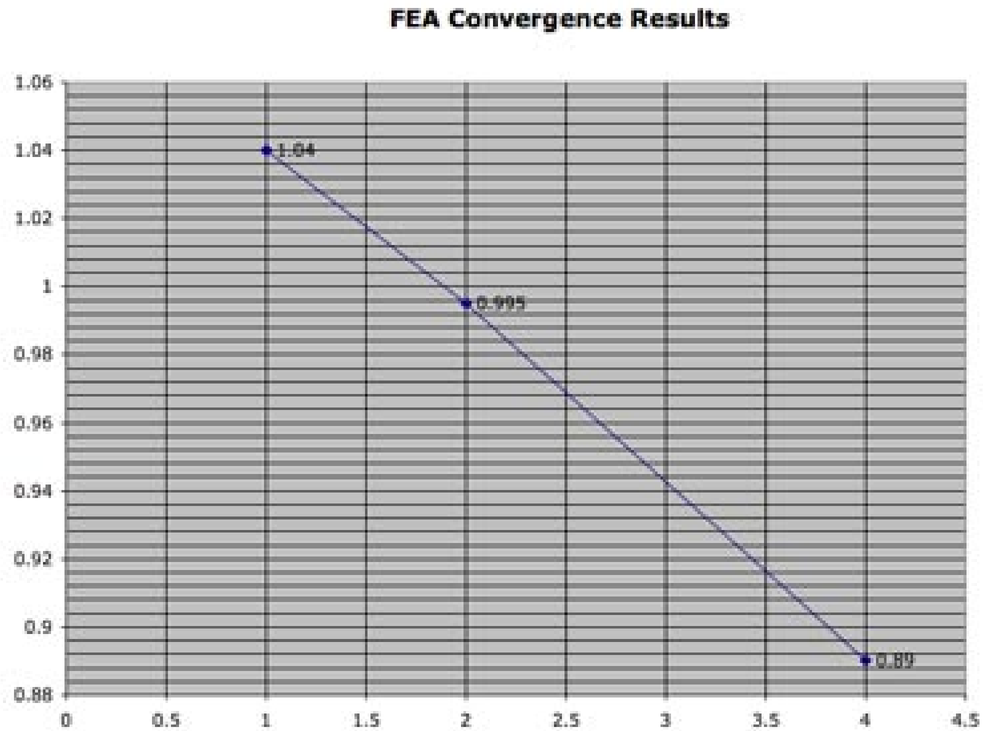
Modified
4mm Mesh

Max Stress = 0.947 N/mm2

Discussion of Modified Mesh

The modified mesh with a mesh point closer to the location of maximum stress returns a significantly higher maximum stress. Compared the maximum stress of the modified mesh of 0.947 N/mm2 to 0.890 N/mm2 for the unmodified mesh (all other parameters equal), the movement of a single node makes a substantial difference in the case of the coarse mesh. This demonstrates that the 4mm mesh is extremely sensitive to the mesh coordinates and indicates that it is too coarse for accurate results.

Results: Convergence



From Class Notes:

$$\sigma_{exact} = \sigma_{FE_1} + c$$

$$\sigma_{exact} = \sigma_{FE_2} + c\left(\frac{1}{2}\right)^n$$

$$\sigma_{exact} = \sigma_{FE_3} + c\left(\frac{1}{4}\right)^n$$

$$set \quad a = \left(\frac{1}{2}\right)^n$$

$$\sigma_{exact} = \sigma_{FE_1} + c$$

$$\sigma_{exact} = \sigma_{FE_2} + ca$$

$$\sigma_{exact} = \sigma_{FE_3} + ca^2$$

$$a = \left[\frac{\sigma_{FE_3} - \sigma_{FE_2}}{\sigma_{FE_2} - \sigma_{FE_1}}\right] \qquad n =$$

$$c = \left(\sigma_{FE_2} - \sigma_{FE_1}\right)/(1 - a)$$

$$and\ hence\ \sigma_{exact} = \sigma_{FE_1} + c$$

Calculation of σ (exact)

a = (1.04 - 0.995)/(0.995-0.890) = 0.429

c = (0.995 - 0.890)/(1-0.429) = 0.184

Therefore σ (exact)=0.890 + 0.184 = 1.07N/mm2

Calculation of Stress Concentration

An applied distributed load of 1 N/mm on a plate 5mm in thickness gives a nominal stress of 0.2 N/mm2.

The maximum stress is extrapolated to be 1.07 N/mm2.

Therefore, the Stress Concentration indicated by the FEA analysis is 1.07/0.2 = 5.35

Next we will compare this result with published results.

Question 3: Comparison of Stress Concentration with Published Results

Published stress concentration results from the ESDU database are referenced:

ESDU #85045 Stress Concentrations:Interaction and Stress Decay for Selected Cases. (Elliptical Hole included.)

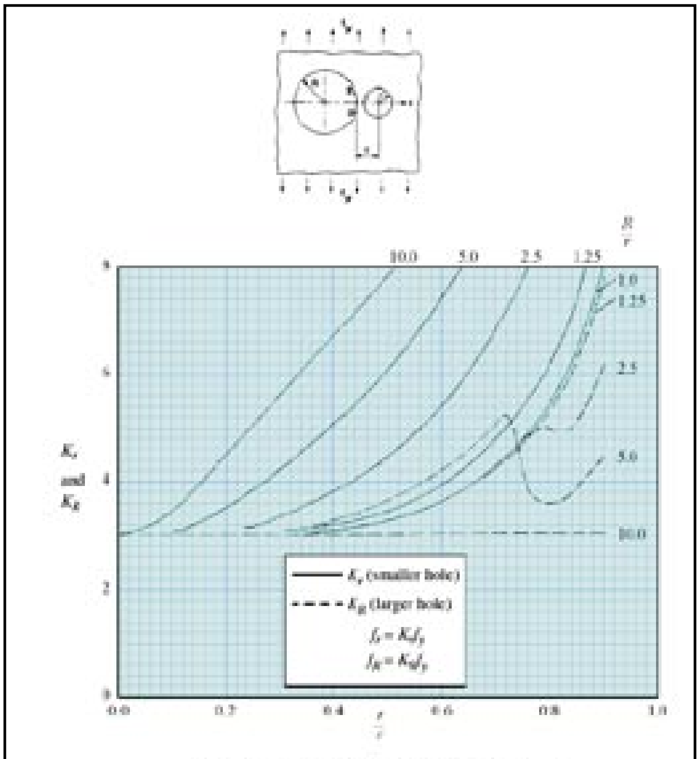


FIGURE 2 α = 0 TENSION IN y-DIRECTION

From ESDU #75007: Geometric Stress Concentration Factors: Two Adjacent Unreinforced Circular Holes in Infinite Flat Plates

Modeling the 30mmx20mm Elliptical hole as a circular hole of 30mm diameter gives us:

R/r= 6

and

r/c = 5mm/(42mm-30m) = 0.42

From the chart we can read form these two values to give us

Kr (smaller hole) = 5.4

Note that since we modeled the ellipse as a circular hole with the larger of the two loci distances, the result is conservative.

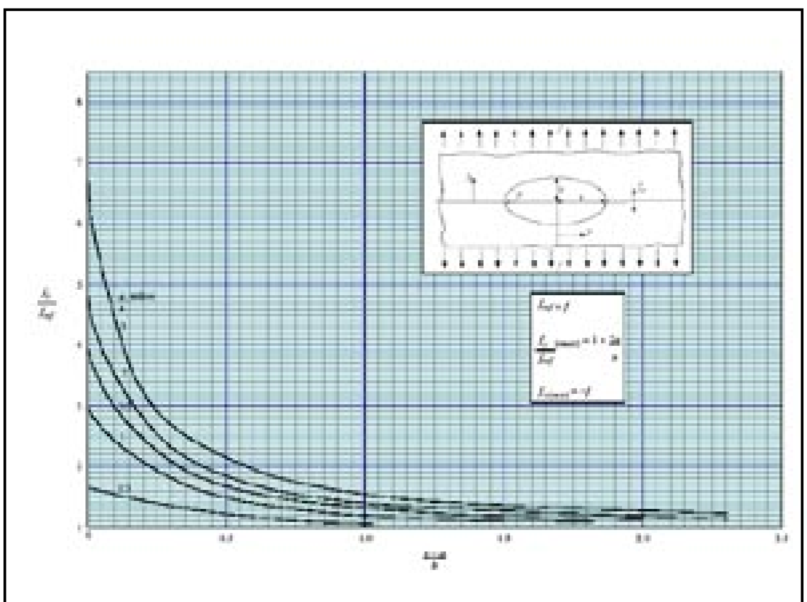


FIGURE 1 DECAY OF STRESS AWAY FROM AN ELLIPTICAL HOLE IN A WIDE PLATE

From ESDU#85045 Stress Concentrations: Interaction & Stress Decay for Selected Cases.

Here we calculate (x-a)/b for the region the smaller circular hole occupies and read values of f_y/f_{ref} with a/b=1.5:

Start of hole:

(x-a)/b =((42-2.5)-30)/20 = 0.32

From chart: f_y/f_{ref} = 2.0

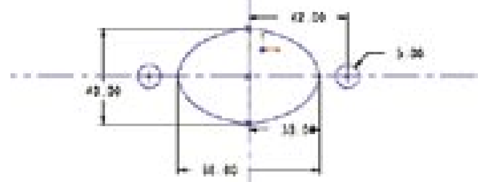
End of hole:

(x-a)/b = ((42+2.5)-30)/20 = 0.73

From chart: f_y/f_{ref} = 1.5

The average of these two values is 1.75, which we use to multiply by the value of the stress concentration of the smaller hole (3.0)

Kt (compound stress riser) = 1.75 x 3.0 = 5.25



CONCLUSION OF QUESTION 3: Stress Concentration

Our FEA analysis indicating a stress concentration of 5.35 comes quite close to the two published results (5.4 and 5.25).

Question 4: Estimation of Fatigue Life of Panel

Using ESDU data sheets #E.07.03 (The Effect of Mean Stress on the Endurance of Aluminum Alloys) and #70016 (Terms and Notation for Fatigue Endurance Data) to estimate the fatigue life of the panel with a cyclical load of 40MPa.

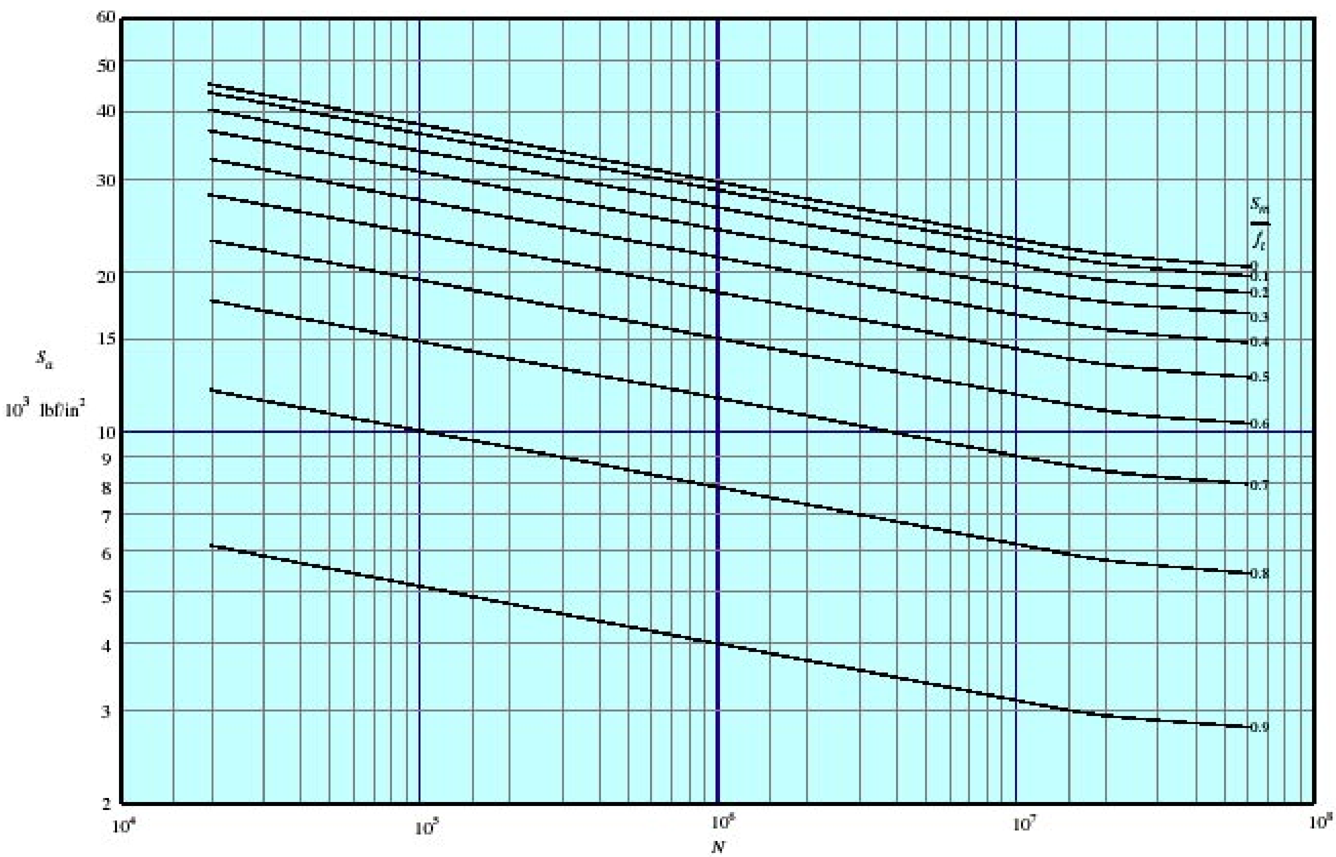
Although we do not know the total stress concentration factor which, in endurance calculations, includes terms involving temperature fluctuations, surface finish, and material composition, we will assume the total stress concentration is given as our previous result of 5.35 and an aluminum such as 6061-T6 or 2024-T4 with an ultimate tensile strength of 446Pa.

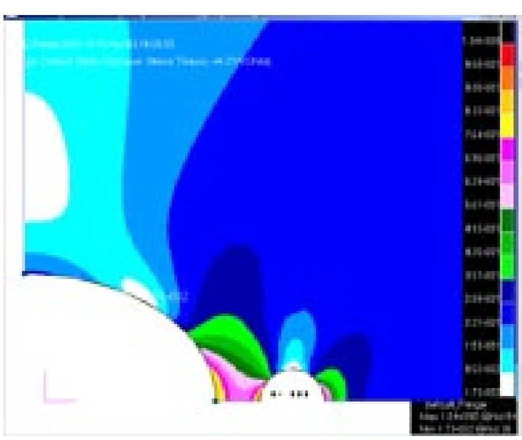
Using conversion factor of 6.89 MPA = 1 ksi,

S_a = (40MPa x 5.35) / 6.89 = 31.1 ksi

S_m/f_t = (20MPa x 5.35)/446MPa = 0.24

From the chart, this gives us Endurance Limit of approximately 3 x10^4 cycles.





Conclusion: The compounding of the stress concentrations of the two holes is computed to be 5.35 the nominal stress on the plate and is located at the centerline edge of the smaller hole The results indicate that the distance between the two holes should be increased for a reasonable design. In addition, the design is prone to failure from fluctuating stresses due to its low endurance limit