EMT 625 AT1 Teach Mathematics to Peers: The Black Scholes Equation

For this assignment, I chose to present information about the Black Scholes equation, which is a mathematical expression for pricing options. It was developed by Fisher Black and Myron Scholes in 1973 (for which they received a Nobel Prize in 1997), and priced options based on the model of perfect markets with random fluctuations about a stock's "rational price". The equation was independent of the expected future value of stocks, and the primary unknown variable was the expected future volatility of the stock. Stocks with a high volatility meant that the option price would be higher, as the probability for a greater return was higher. By selling short and buying options, investors could create "risk-free" portfolios, enhanced by taking advantage of under priced options.

For my presentation, I began by discussing the concepts of selling short, and the purpose of options, and presented a graph which illustrated the limit to losses (yet unlimited profits) that were possible with the purchase of an option. I then presented the partial differential Black Scholes equation and discussed the variables involved with the equation.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0.$$

After considering the variables, I discussed volatility (sigma σ) in terms of the underlying model based on Brownian Motion (the stochastic diffusion of particles discovered by the botanist Robert Brown in 1827) and described theoretically by Albert Einstein in 1905, who determined that the mean displacement was proportional to the square root of time. The mathematical principles of Brownian Motion were experimentally verified by Jean Baptiste Perrin in 1909, for which he received the Nobel Prize in 1926 (his work enabled the calculation of Avogadro's number and was considered the first "proof" of the existence of molecules). Based on earlier work by Louis Bachelier and others, it was also found that stock markets also followed this stochastic pattern (when markets were acting "rationally").

The Black Scholes equation has been in widespread use since its inception, but despite caveats from Black and Scholes of the underlying assumptions, continued to be utilised in times of market uncertainty, and has been considered responsible for several major stock market crashes. A "Black Swan" is a term used to represent improbable occurrences (I discussed the history of the term in my presentation), where the market fails to act rationally, and are more common in the stock market than the "perfect market" model predicts.

I discussed the "perfect storm" that brewed in the 1970's, when the Chicago Options Exchanged opened, allowing public trades in options for the first time in the US, combined with Black Scholes equation and pocket calculators which provided widespread access to the ability to compute options pricing. In the absence of "Black Swan" episodes in the market, the equation was solid, but when the "madness of the crowds" occurs (in contrast to the normal "wisdom of the crowds"), the equation fails due to the erroneous predicted value of volatility.

The talk could be a lead in for a further analysis of Gaussian distribution, which describes the statistical probabilities of values based on their variance. The lesson can include the consequences of using such a model in the presence of outliers, and examine additional models of statistical inference and other aspects of probability theory.

Primary source: *Pricing the Future, by George Szpiro, 2011, Basic Books, NY.*

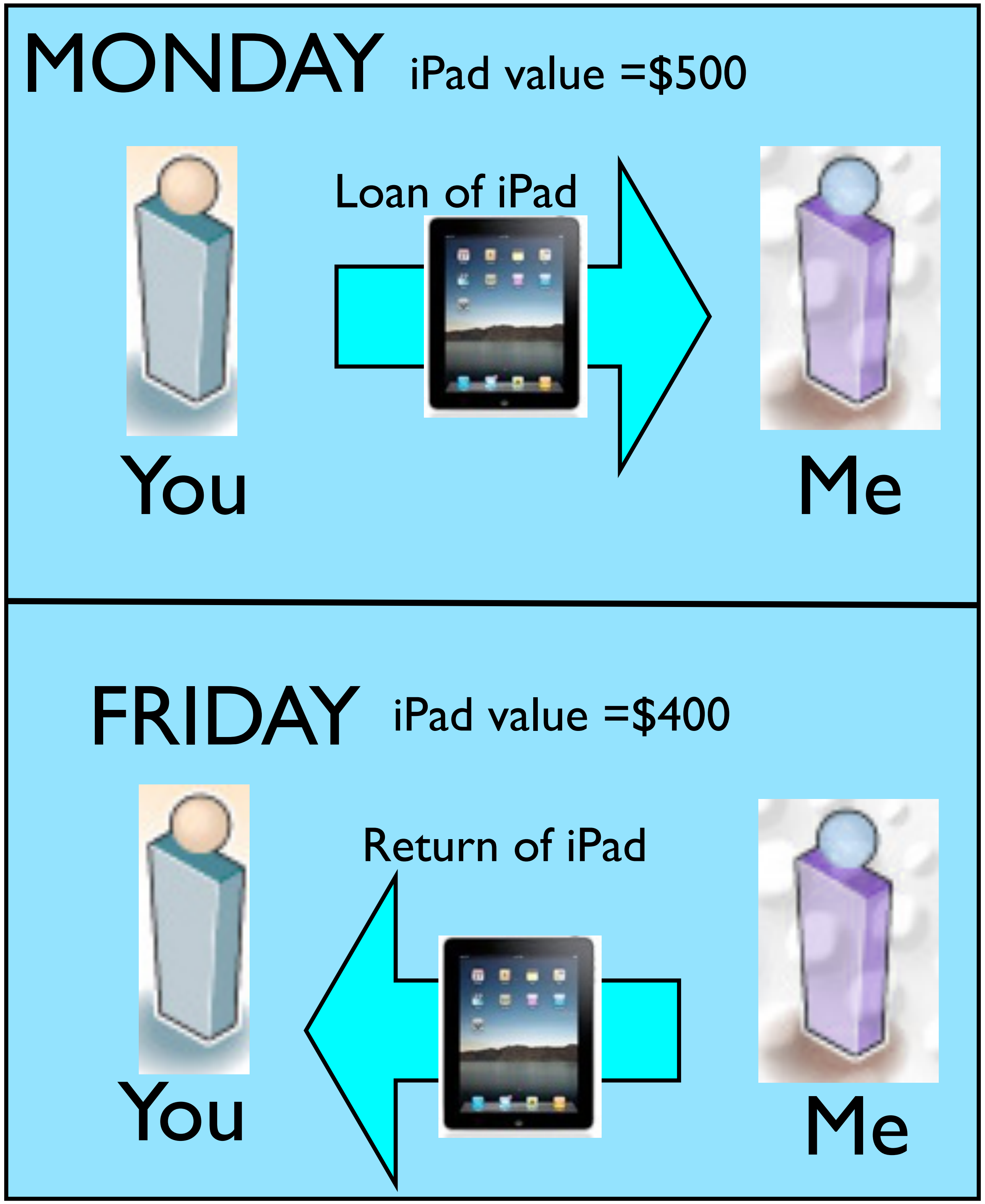
“How much food are we talking about?

The Rise and Fall of a Nobel Prize Winning Equation

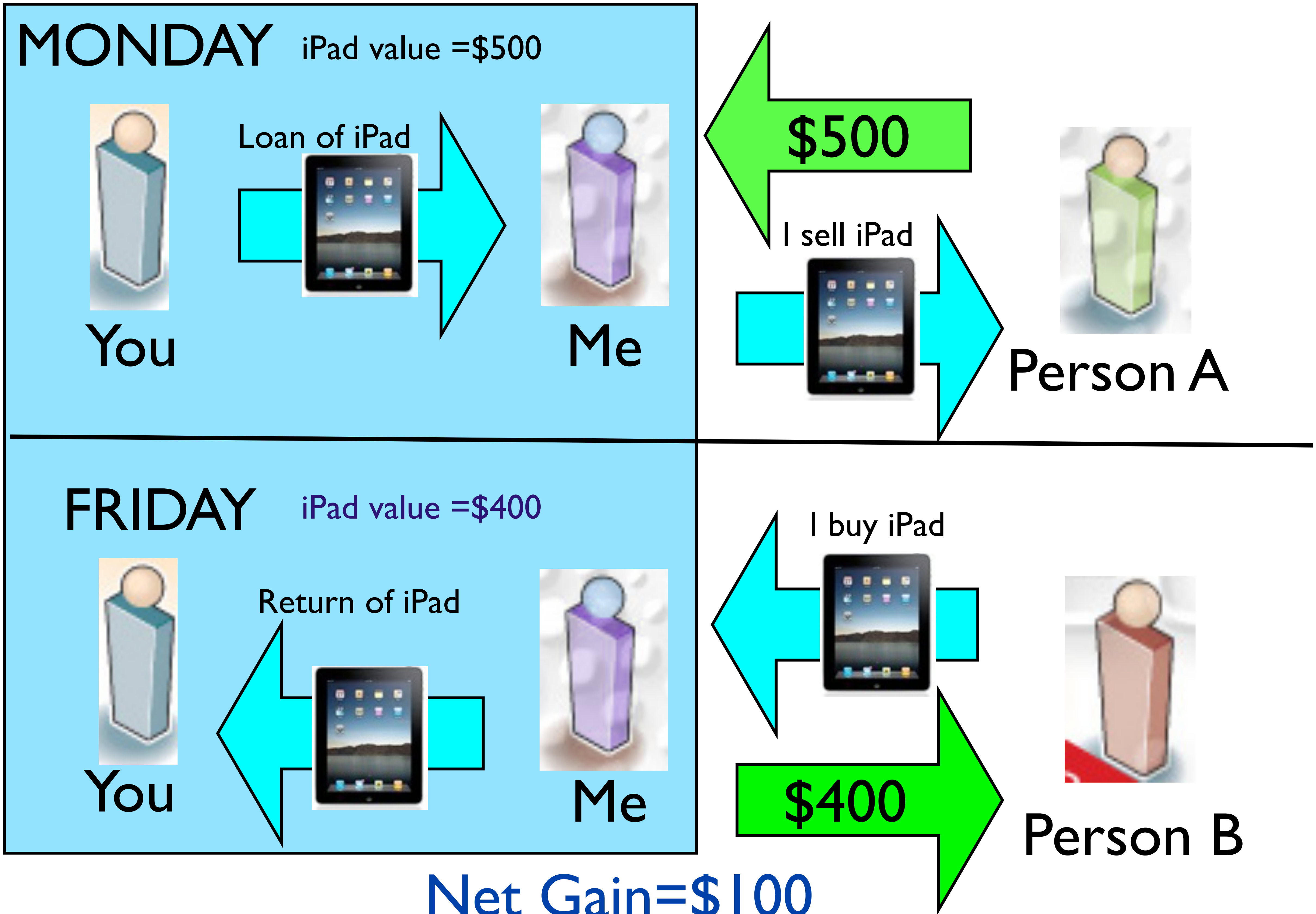
$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0.$$

The Black Scholes Equation

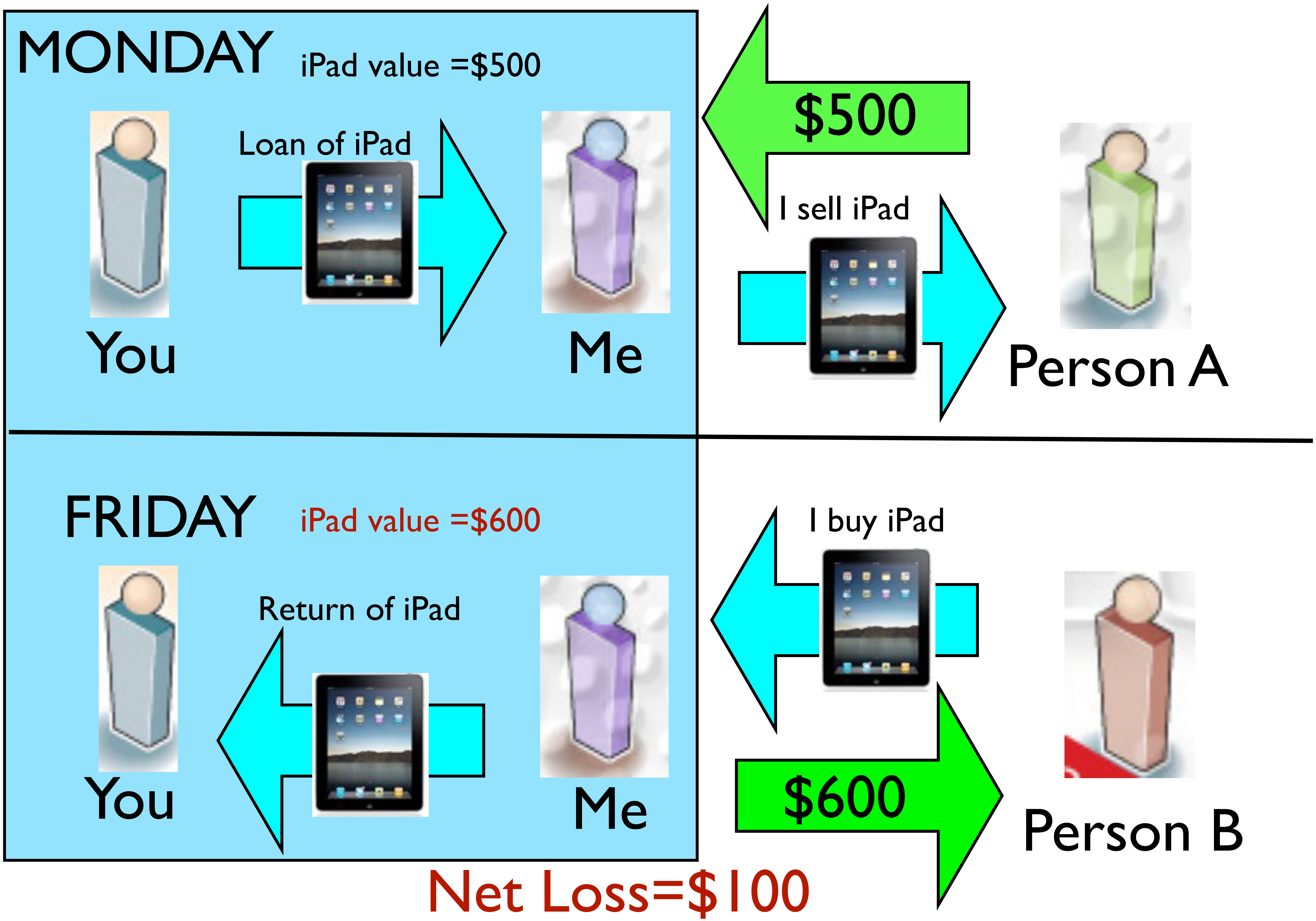
How would an investor make money here?



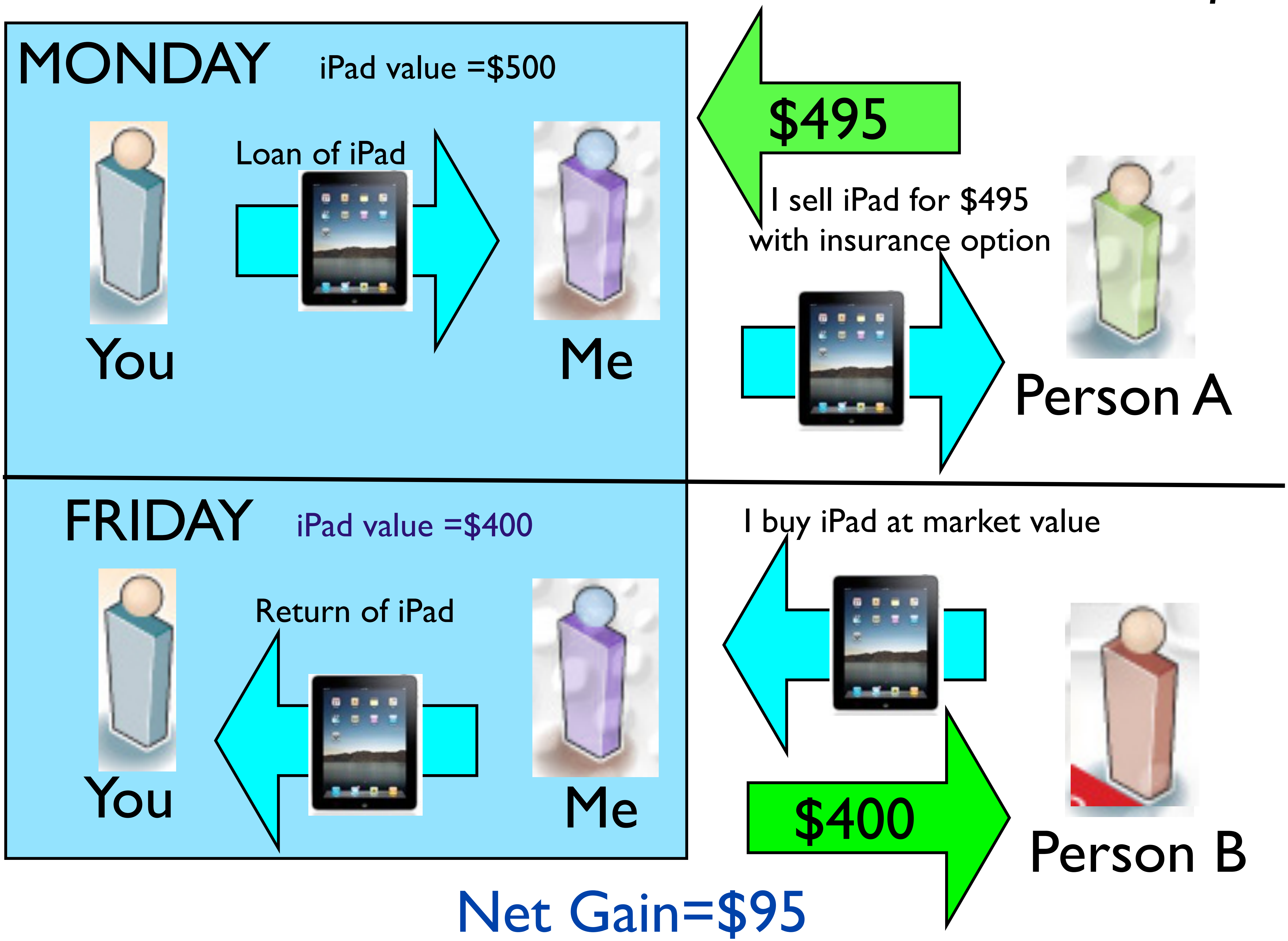
SELLING SHORT--iPad value decreases



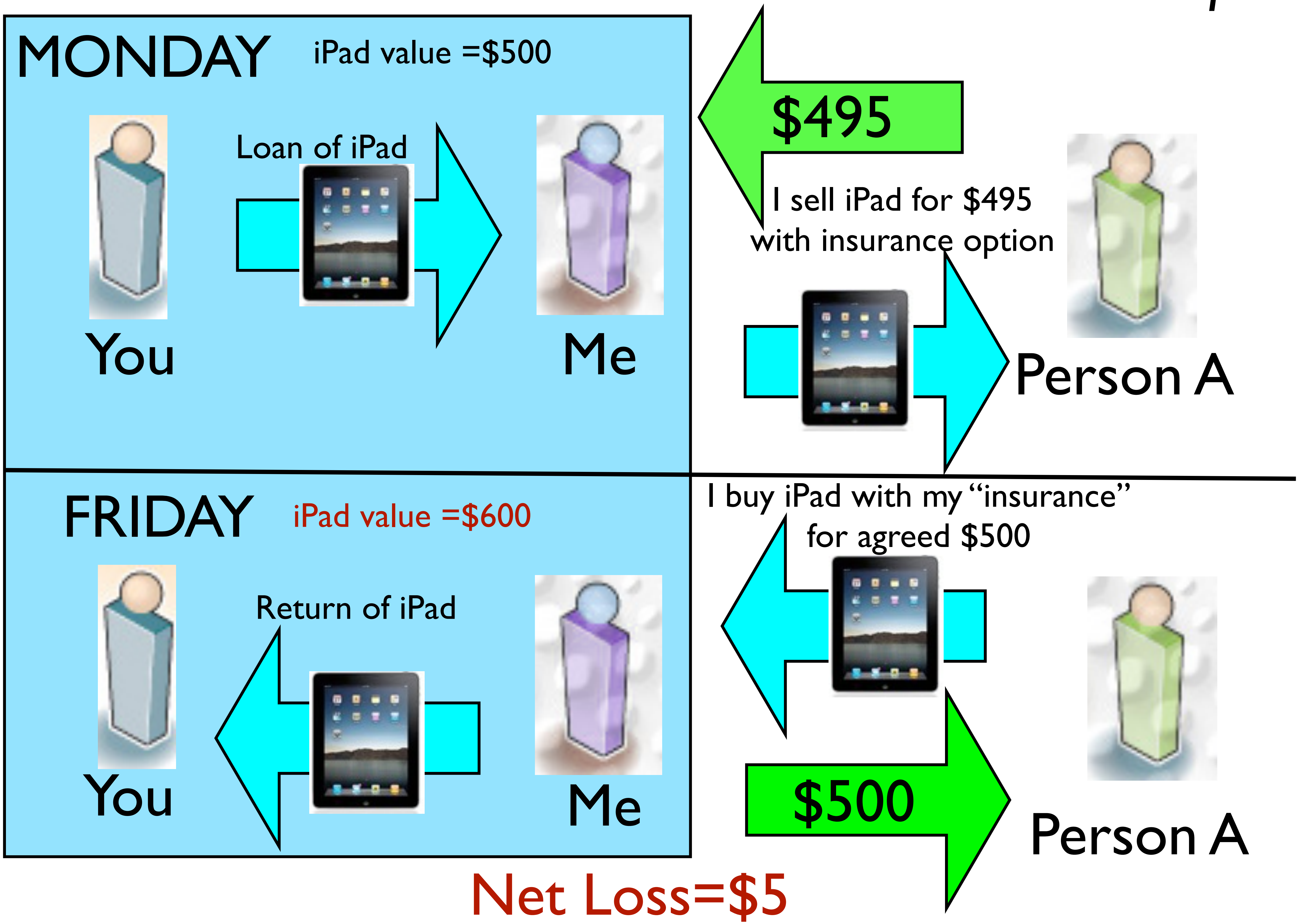
SELLING SHORT--iPad value increases



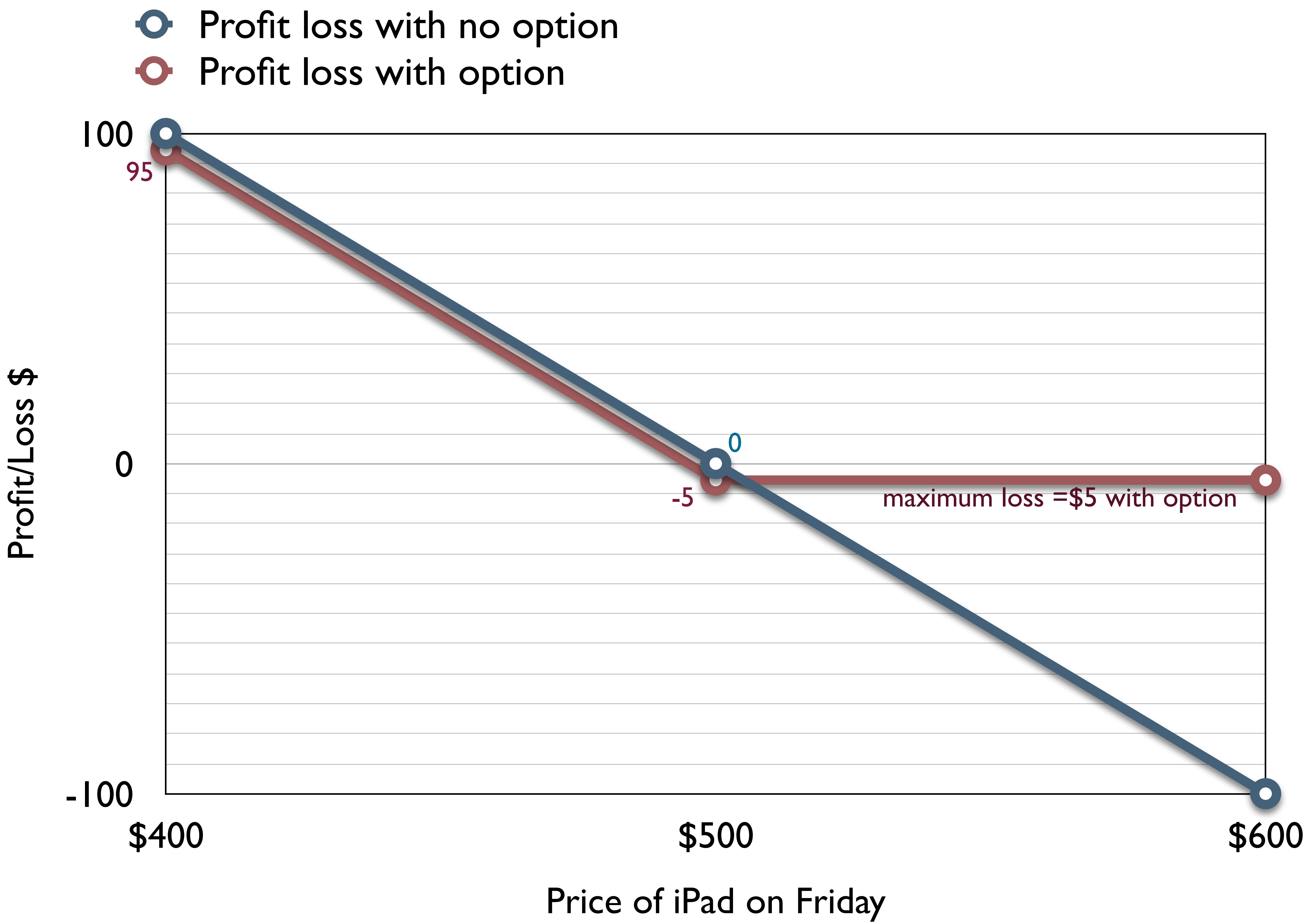
SELLING SHORT--value decreases with insurance option



SELLING SHORT--value increases with insurance option



Selling Short (with or without Option "Insurance")



Enter Fisher Black and Myron Scholes

Source: http://bradley.bradley.edu/~arr/bsm/model.html

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0.$$

S=price of stock
V=price of option
r=risk free interest rate
σ=volatility
t=time

Price of an option dependent on:

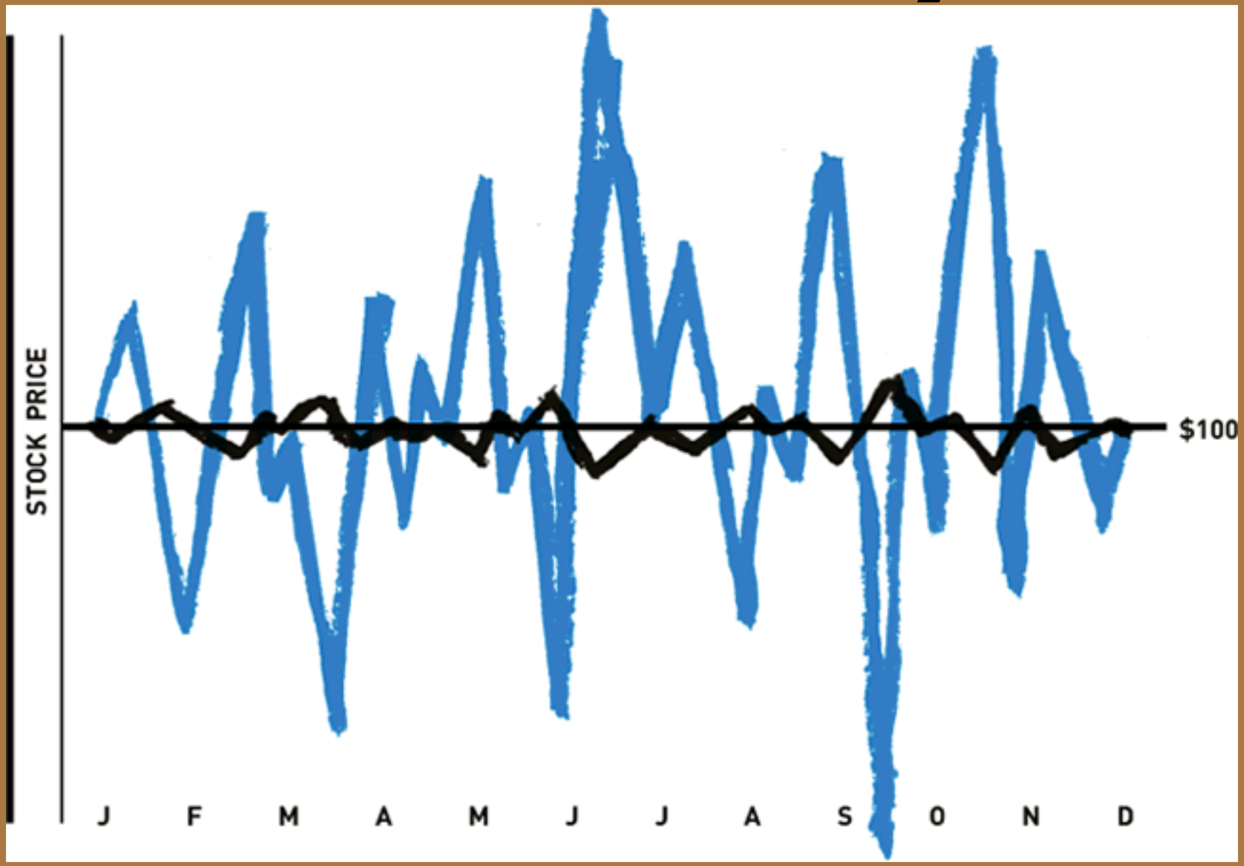
- current stock price (a known value)
- bank interest rate (the known fixed cost of borrowing money)
- time (the known time to return "loan" and potential exercise of option)
- stock volatility (σ) over time of contract (unknown!)

1973--the perfect storm brews

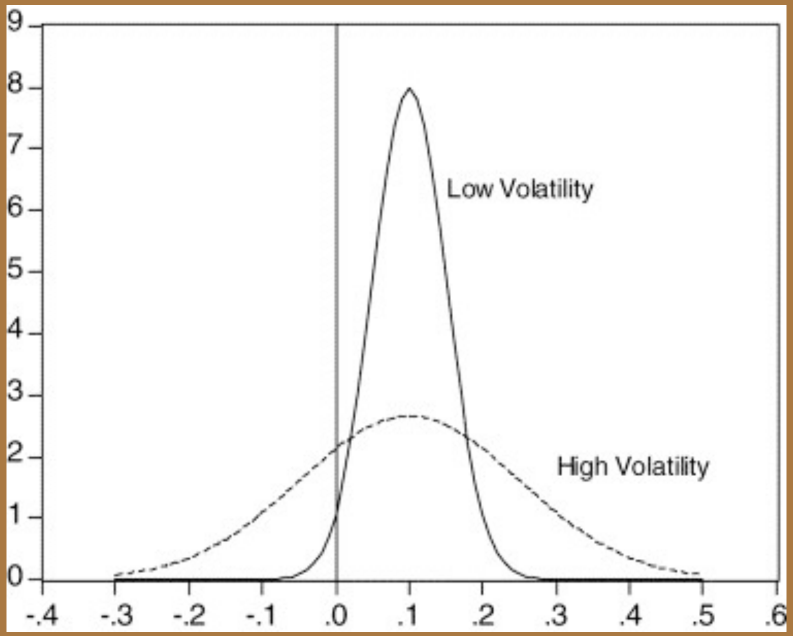
CBOE
CHICAGO BOARD OPTIONS EXCHANGE

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0.$$

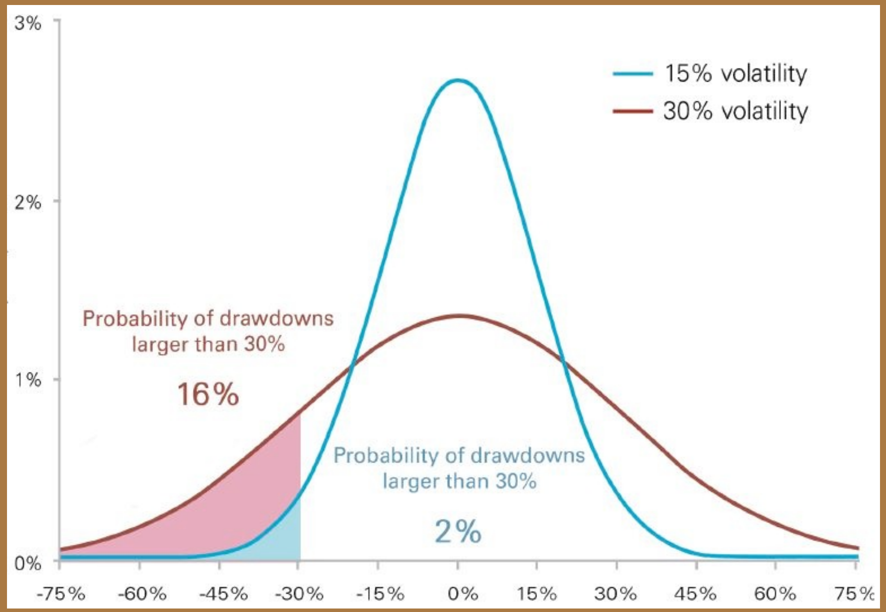
Volatility



STOCK PRICE
$100
J
F
M
A
M
J
J
A
S
O
N
D

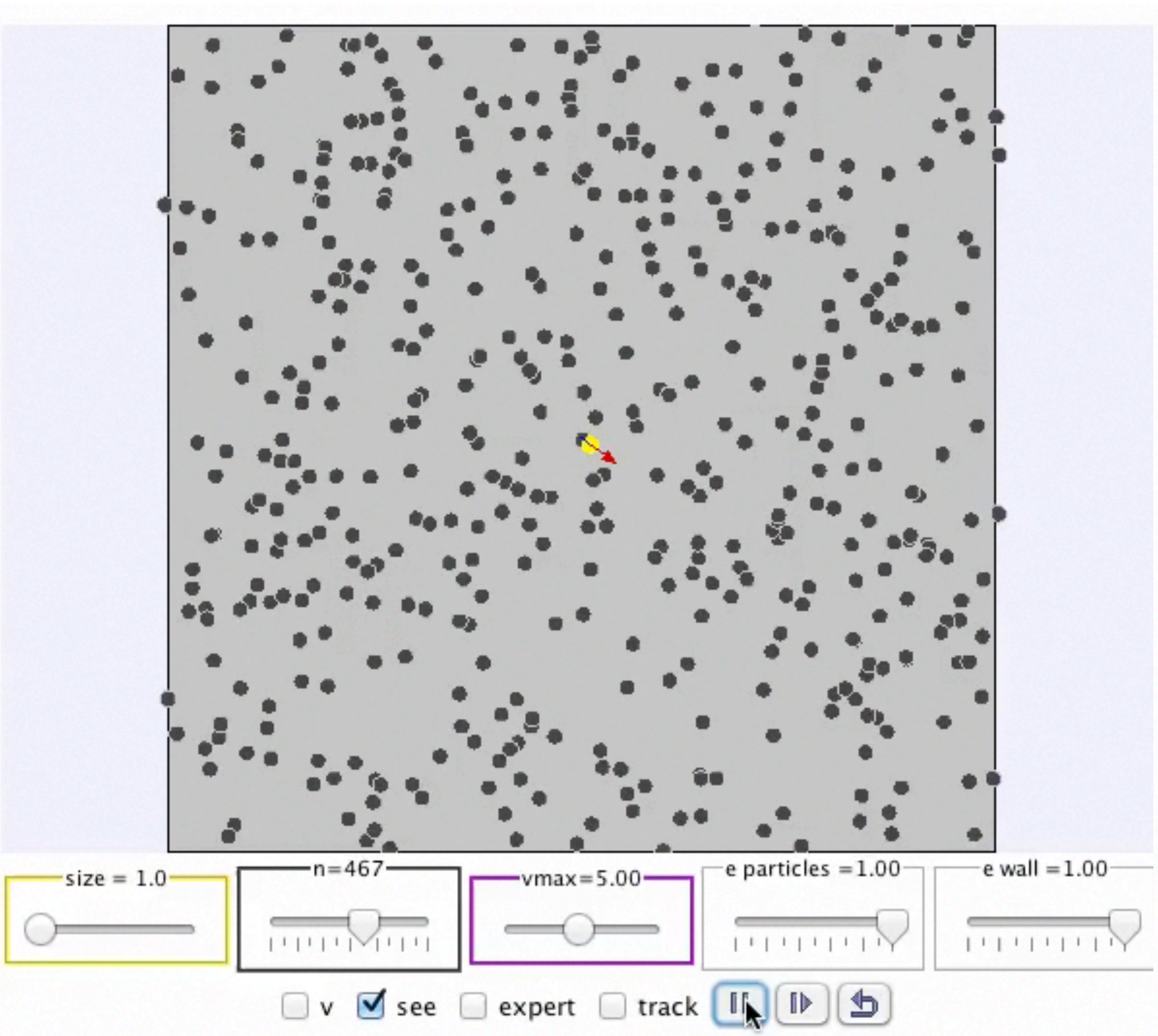


Low Volatility
High Volatility
-.4
-.3
-.2
-.1
.0
.1
.2
.3
.4
.5
.6

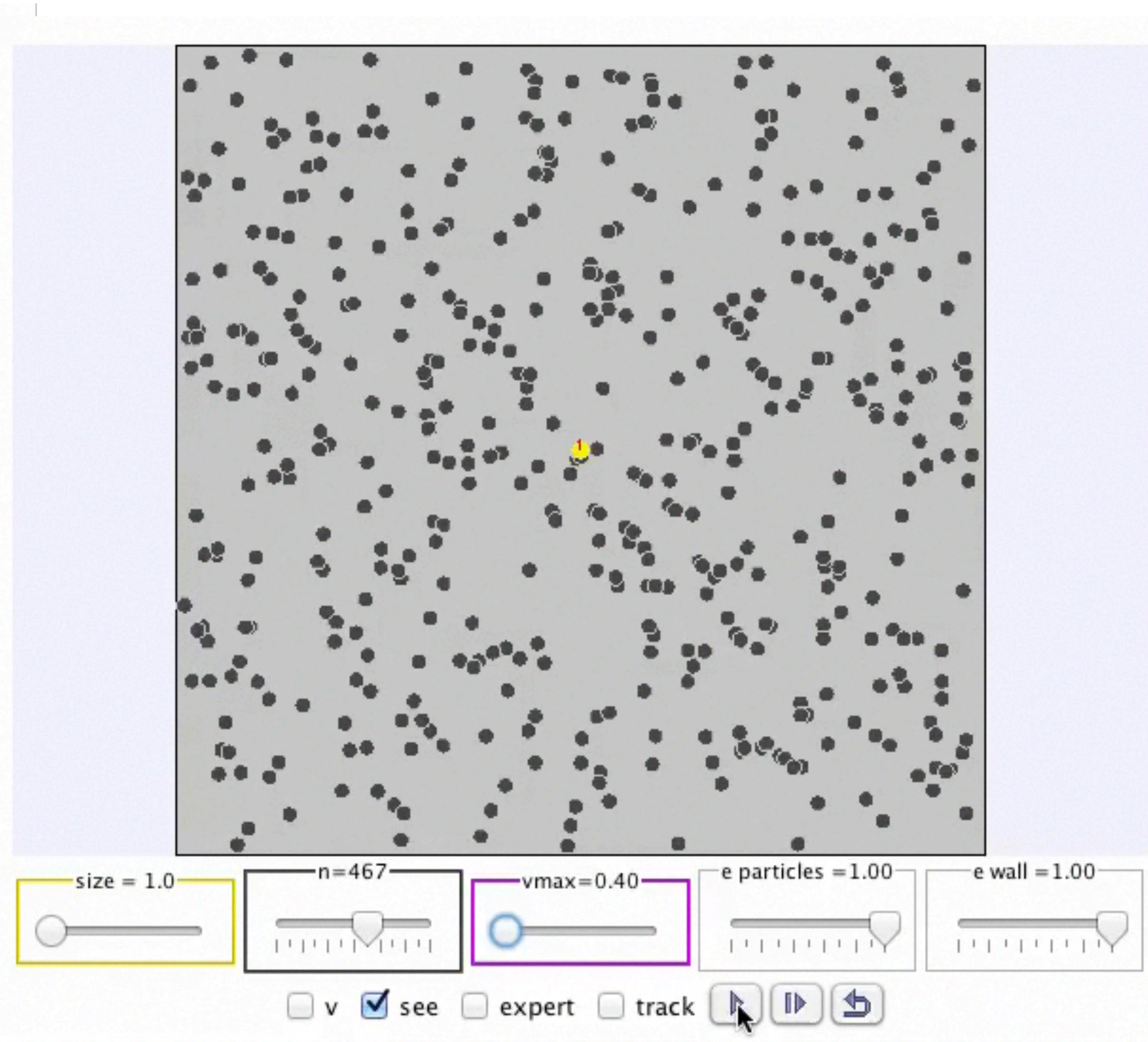


15% volatility
30% volatility
Probability of drawdowns larger than 30%
16%
Probability of drawdowns larger than 30%
2%
3%
2%
1%
0%
-75%
-60%
-45%
-30%
-15%
0%
15%
30%
45%
60%
75%

BROWNIAN MOTION



Hot (high "volatility")



Cold (low "volatility")

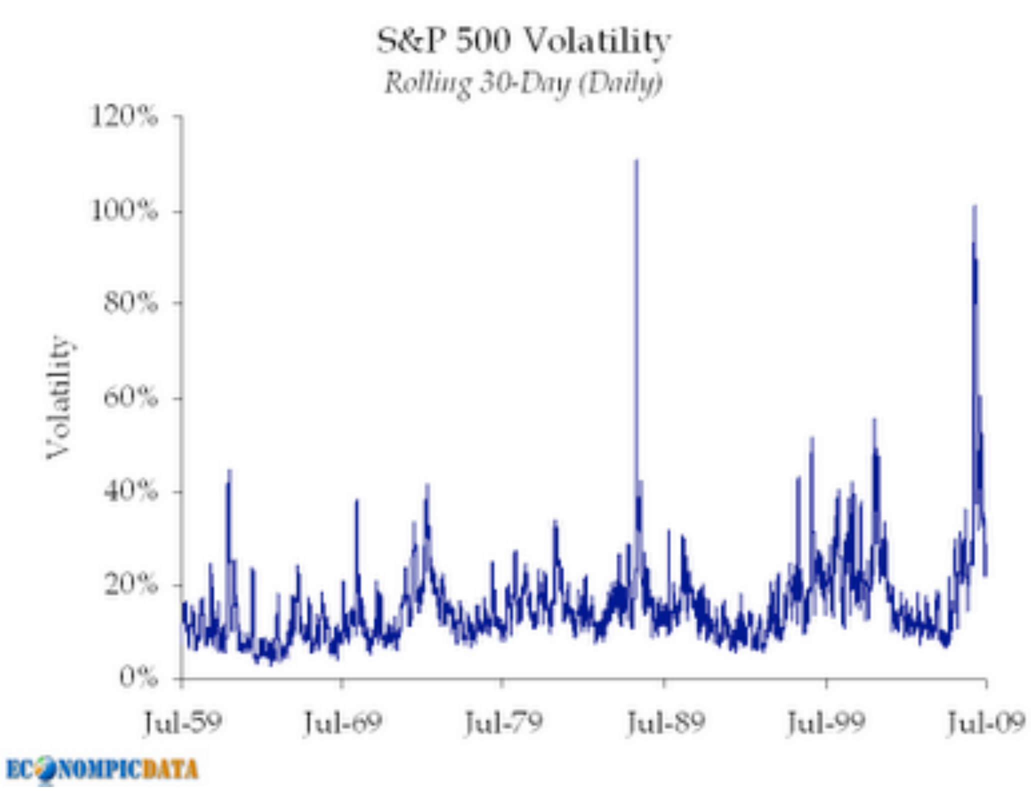
Black-Scholes Key Assumptions:

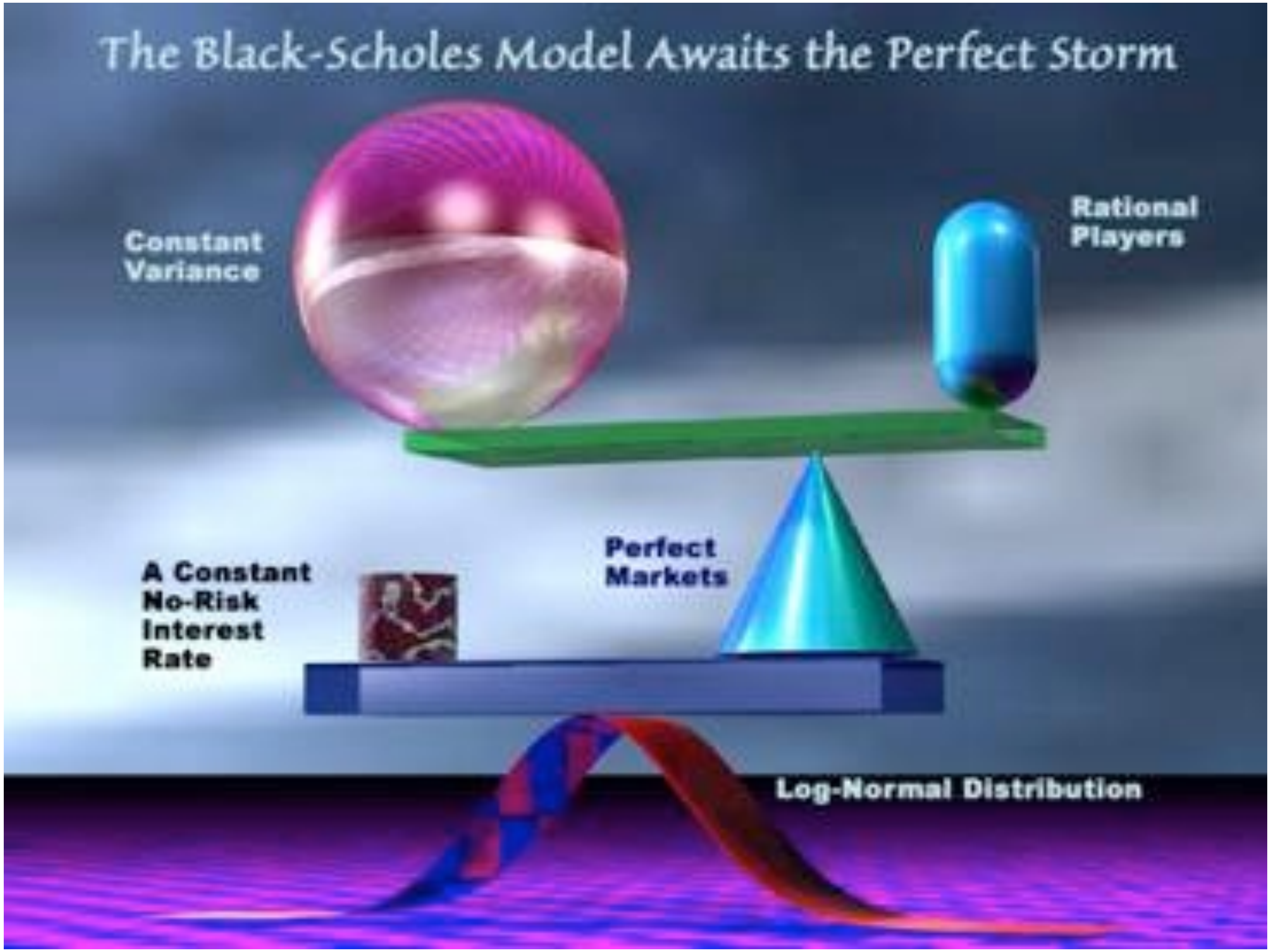
- “perfect” markets with random fluctuations
- constant volatility over time of the option contract

Standard Deviation
Over 10 Year Period Ending 1/31/04

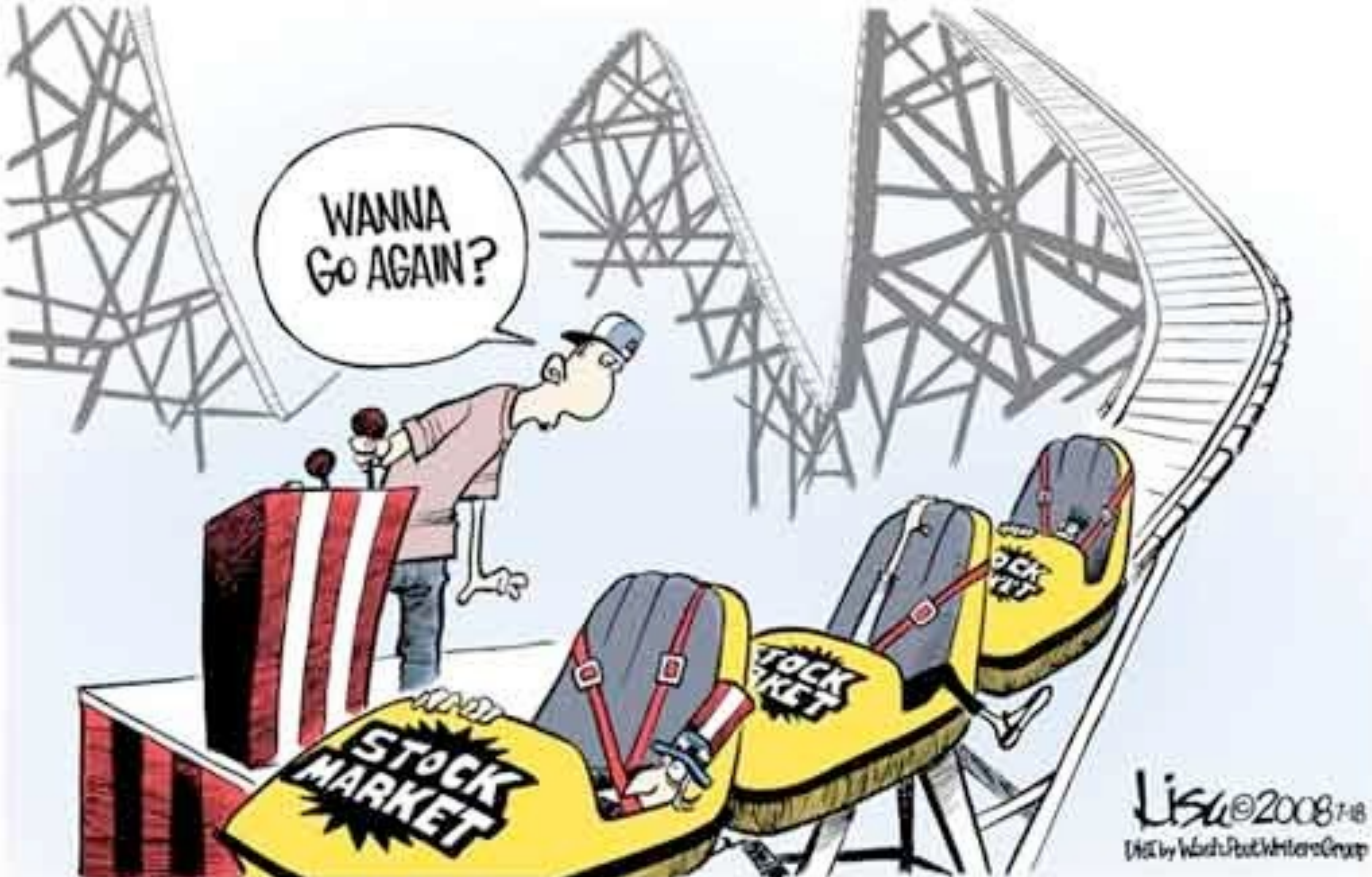
At Interval:	**S&P 500**	**NASDAQ**
Daily	1.1%	1.8%
Weekly	2.4%	3.8%
Monthly	4.5%	8.3%

$$\sigma_T = \sigma\sqrt{T}.$$





source: http://www.capital-flow-analysis.com/investment-essays/nobel_gods3.html


WANNA GO AGAIN?
STOCK MARKET
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