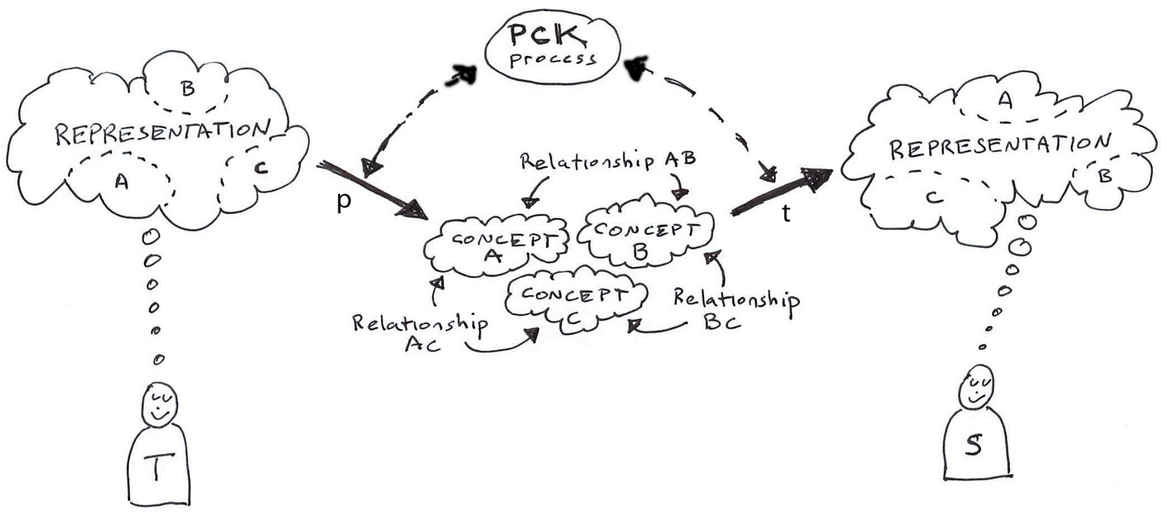
PCK Analysis of a Teaching Episode

Teaching is a complex profession, marrying knowledge with pedagogy, the craft of instruction. The classical teacher Socrates exemplified teaching as a conversational dialogue between teacher and student, and differentiated deductive reasoning with inductive reasoning; deduction (premises logically leading to a conclusion) constructs cognitive learning, while induction (validation of a conclusion with premises) either supports or exposes flaws in learning. When purposefully applied, both reasoning techniques are critical informants of a teacher's content and pedagogical knowledge, and are exposed in the two fundamental "Socratic scaffolding" tools of the teacher: questioning and interpreting, the latter in the form of explanation and presentation of activities.

Shulman (1986) identifies pedagogical content knowledge (PCK) as knowledge that "goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching" (1986, p.9), and the ways teachers represent and formulate content "to make it comprehensible to others" (1986, p.9). The ultimate test of understanding "rests on the ability to transform one's knowledge into teaching. Those who can, do. Those who understand, teach" (Shulman, 1986). Shulman goes on to classify various types of teacher knowledge, and differentiates teacher *strategic knowledge* applied when "principles collide and no simple solution is possible" (1986, p.18); for example, the seemingly incompatibility of the positive effects of wait time on higher-level cognitive processing (Rowe, 1974, as cited in Shulman, 1986) with the negative effects of a slow pace on classroom behaviour (Kounin, 1970, as cited in Shulman, 1986). It is a teacher's strategic knowledge--knowing the how, when, where, and why to apply principles--that is the heart of effectively applied PCK.

An analysis of PCK in the classroom can help measure the effectiveness of teaching--the teacher behaviours and strategies that lead to achievement gains among students. Shulman (1986) especially emphasises the importance of understanding prior knowledge and preconceptions of students of different ages, backgrounds and abilities; any discussion of PCK, therefore, must be grounded in the context of the learning environment. Shulman is explicitly constructivist in his analysis--he describes teaching as the exchange of "shaped and tailored" ideas (1987, p. 13), informed by teacher "comprehension and reasoning, as transformation and reflection" (1987, p.13), which are then actively grasped by the student beyond imitative comprehension; in other words, involving both cognitive and affective changes within the student. This notion can be elaborated further with 'ideas' recognised as cognitive representations that can be deconstructed into appropriate atomic components, which are then presented to the student in order to be cognitively reconstructed. This concept is illustrated below:



Above: By first examining how a cognitive representation might be variously reconstructed from its constituent concepts, the teacher deconstructs a representation, then presents (often reframing) individual concepts--and their relationships--so student can assimilate and accommodate. Though possibly structured differently, both teacher and student representations result in similar conclusions from a given set of input premises. Planning takes place at arrow marked 'p' and teaching takes place at arrow marked 't'. Note that the teacher's cognitive representation above is a subset of teacher's broader knowledge (and representations) and appropriately informed by the specific teaching/learning context.

With these three concepts in mind: Socratic scaffolding, the application of strategic teacher knowledge, and the atomic presentation of conceptual representations, we will examine the PCK of an exemplary year 8 lesson on Pythagoras' theorem (ProTeachersVideo, 2012); the lesson outline is reconstructed in Appendix 1. The focus will be the effectiveness of the lesson's objectives based on the teaching approach (informed by an analysis of conceptual representations), engagement (informed by strategic teacher knowledge), and communication (informed by Socratic scaffolding techniques); together they support the design sequence of achieving outcomes in the classroom: first, consider the conceptual cognition of the students to plan the teaching approach; second, incorporate activities to engage the students; third, prepare probing questions that scaffold cognitive construction deductively, build on alternate conceptions inductively, and test for understanding with communication.

Teaching Approach

The teaching approach, and how it relates to the atomic presentation of conceptual representations, can be discussed in terms of two cognitive linkages: the concept/language link, and the formal/informal knowledge link. Dan Walton, the teacher in the video, has structured this constructivist inquiry-based lesson with a "mathematical concept first, terminology second" approach; remarkably, though he is teaching Pythagoras, not once does he introduce the nomenclature “hypotenuse”. Likewise, the Pythagoras formula ($a^2 + b^2 = c^2$) is never explicitly provided. The outcomes could be stated:

•*Students can discover and describe use of Pythagoras theorem using student language.*

•*Student can identify and contrast two forms of Pythagoras problems (SS-->H & SH-->S).*

•*Students can apply Pythagoras theorem to calculate third side of right-angle triangles.*

Dan’s aim is to “get them to discover the concept” (ProTeachersVideo, 2012). He does this by having one group solve a numerical problem (not yet known to be linked to Pythagoras), and one group solve a graphical problem. The graphical solvers use prior graphing knowledge to empirically determine the length of the hypotenuse of a 6-8-10 right angle triangle with a ruler; the numerical solvers tackle a problem (presumably based on prior challenges presented in class) of “finding an answer of 10” from a cloud of symbols (6, 8, +, =, x[‘times’], √ [square root]). All the graphical solvers arrive at 10, while only three students solve the numerical challenge. Dan notes that though only a few students derived the numerical method, the graphical solvers are “itching” to hear the numerical method presumably due to the similarity of the variables 6, 8, 10 in both problems (this would also apply to the unsuccessful numerical solvers, though they would not had the same direct cognitive benefit from working the graphical solution). Dan then picks one of the successful numerical solvers to provide a verbal description of the solution, which becomes the class Pythagoras method. In pairs, students then “machine gun” the student language method using specific examples (“six times six, eight times eight, add, square root”) in a playful timed game which enables both self- and peer-assessment. Dan also uses student language for other concepts, such as “square root it”, when discussing taking the square of a number.

The student language approach is discussed by Herbel-Eisenmann (2002), who terms it “Contextual Language” (CL), who notes that is can be “a base on which to build a connection to formal mathematical language” (p.1). It is very successful approach in Dan’s class, though Herbel-Eisenmann emphasises the importance of moving the students “from using their own language to using words that are more mathematically appropriate” (p.1). In order to secure the student’s mathematical representation of Pythagoras, it would be advantageous to further scaffold the official mathematical language (e.g. hypotenuse) in subsequent lessons in order to enhance subsequent communication of the representation and to strengthen the student’s language link to the concept.

The second PCK teaching approach Dan uses effectively is the scaffolding of informal (“everyday”) mathematical knowledge. He does this by introducing an authentic scenario-

-a dogleg golf hole illustrated by a large whiteboard image. It is intuitively apparent that the diagonal of the right angle triangle will be shorter than the overall length of the two sides, or, as Dan puts it, the "direct route". This use of informal knowledge is very effective and provides a representational link to the corresponding formal knowledge of the properties of a hypotenuse (i.e. *shorter* than the combined length of the two orthogonal sides). Russell & Ginsburg (1984) note that children who present mathematical difficulties generally have "adequate informal knowledge of mathematics" (p. 240); thus, building on informal knowledge enhances the success of a greater number of students to achieve the cognitive representational outcomes of the lesson. To prevent any alternate conceptions, a PCK consideration would be to consider scaffolding the "direct route" as *longer* than either of the two shorter sides (though this becomes apparent after the student's numerical calculation of the golf dogleg diagonal); this could then be conceptually reinforced in the later identification of the "two types" of Pythagoras problems.

After solving the golf problem and identification of the two types of Pythagoras problems using a "spot the difference" activity and scaffolding its variant method using a provided "help sheet" (Appendix 1), most students appear to have a cognitive representation of Pythagoras theorem through Dan's provision of the carefully sequenced atomic concepts, and are prepared for the final consolidating activity. In this sense, the affective outcomes are met--the students have organised and internalised the conceptual aspects of the Pythagoras theorem. Similar to progression from student language to formal language, these informal concepts can be further reinforced with formal mathematical representation in order to further secure the cognitive representation of Pythagoras' theorem in a way that will be advantageous as complexity increases (e.g. Pythagoras's theorem in three dimensional problems).

Engagement

Dan's approach to engagement is enviable--the number of activities covered in a single lesson is remarkable, and behaviour issues seem to be absent. He begins the class in a structured way by controlling student's entry into class, then immediately engages the students with an activity (flipping a coin or ruler to determine groups). Without pause, the students are instructed on the next activity, provided a minute, and initiated with an enthusiastic "Go!". The rapid pace of instruction alternating with timed activities continues for the entire lesson, cumulating in a kinaesthetic outdoor "treasure hunt" involving prior knowledge (grid locations).

During the lesson's six engaging activities (see Appendix 1), a pattern emerges of alternating detail work (e.g. the development and application of Pythagoras' theorem), and overview work (e.g. paired brainstorming of the dogleg diagonal computation). The ensuing engagement from this varied pace is well aligned with the outcomes of the lesson, with each activity authentically linked to the golf dogleg and sequenced to reinforce the lesson outcomes and cognitive representation. The authenticity enhances engagement (Marks, 2000) and the engagement is further enhanced by the active and collaborative tasks (Zhao and Kou, 2004).

Dan's strategic knowledge clearly informs the timing and pace of his lesson, and is leveraged by knowing the students and their abilities, as well as fundamental human psychology. For example, Dan times his plenary after the excitement of the consolidating game but prior to announcing its winners and awarding prizes, thus ensuring continued engagement for the lesson review. During the review, several students sum up the lesson and explained the learning, which is reiterated and further refined by Dan, who then finally reveals the lesson outcomes: to solve two types of right angle triangle problems using Pythagoras' theorem.

Communication

Communication ties all the elements of teaching together, relies heavily on teacher strategic knowledge, and is the key element in both the exchange of ideas and checking for understanding. In the lesson, the cognitive representation of various Pythagoras problems is tested with a premise and a response from students, and Dan uses deductive stretching and inductive guiding effectively. The questions asked are often open-ended and encourage a thoughtful answer. Dan uses wait time effectively (Lemov, 2010), and provides a timed discussion period for the trickier question in order to help students take ownership of the question. More specific to PCK are the times when he is guiding cognition: by using "we" language, he invites the class to problem solve and deductively builds on more refined answers during their responses, for example, when he is extending student responses about the nature of the golf diagonal being the shortest path. He also is effective with incomplete student responses, when he constructively agrees, but adds, "But I will argue, that..." and continues with scaffolding new ideas. His use of non-verbal communication is also very effective, for example when he looks puzzled after posing a problem, and the way he represents the two types of Pythagoras problems with arm symbols.

Communication is used in a variety of additional ways; for example, the enthusiastic process praise that Dan uses ("I'm impressed", "perfect answer", "Boiling! Get the sun cream") builds self-esteem and encourages a growth mindset (Dweck, 2006). Communication is also effectively used as a cue for guiding activities, such as the "Have a chat" when initiating pair work, or "your challenge, ..." when posing a problem for the students to solve. The only evidence of direct instruction in the entire lesson is the point where he tells students that the additive Pythagoras method will "not get the right answer", followed by the scaffolding of the correct method. Overall, Dan is clear with instructions and indeed, the lesson is a dialogue between teacher and students, perhaps best exemplified when Clare is describing the method, during which he step-by-step reiterates the method returning to Clare with a hearty "Back to Clare."

Conclusion

What becomes clear from an analysis of PCK of an exemplary constructivist lesson is that PCK is not the simplistically sketched Venn diagram of content and pedagogy; rather it can be better symbolised by an equilateral trifecta of content-pedagogy-student. Especially with mathematics, retained knowledge is critical to build further cognition; through the appropriate application of PCK, a teacher can effectively support student's construction of enduring cognitive representations.

Appendix 1: Overview of lesson

Although the lesson video is edited, the whole lesson likely had this outline:

•Engage with coin/ruler toss to split class into two groups.

•Outline challenge for each group and provide one minute work time.

--Group ‘heads’: draw right angle triangle with short sides with lengths 6 and 8. Students find the length of the long side graphically by measurement.

--Group ‘tails’: draw 8,6,+, x (times), =, and the square root sign in ‘cloud’ on whiteboard. Students use numbers and symbols to mathematically construct a way to find 10.

•Introduce golf problem on whiteboard: dogleg hole with right-angled 120m and 70m sides. Discuss purpose of golf and strategies for play. Show golf clubs. Scaffold visualisation of shorter path. In pairs, students discuss how to find distance of hypotenuse.

•Back to first challenge. Measuring group--should all get 10. Numerical method group--ask who got 10; question method--pick student to explain in student language. Note that answer is the same answer as measurement method, with similar numerical input. Write numerical method in student language on board (?--not sure if he did this).

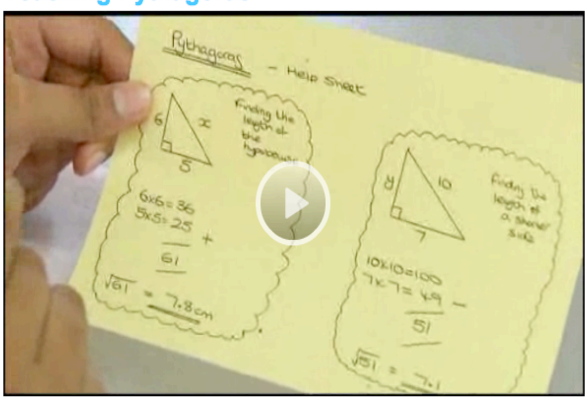
•Provide 90 seconds for golf answer--offer choice of graphical or numerical method. Review answers and scaffold efficiency of numerical method.

•Method game--in pairs, students quickly rattle off student language verbalised method using examples shown on whiteboard, e.g. “six times six, eight times eight, add them together, square rooted”. Self and peer assessment of verbalisation of method.

• Whole class--spot the odd one out game. Provide 4 right angle examples on board, 3 with two short side measurements, 1 with one short and one hypotenuse measurements.

•Provide yellow “help sheet” with “two ways” of Pythagoras and time to review.

Teaching Pythagoras



•In pairs, student consider new problem with hypotenuse and short side--how to find the length of other short side. New method becomes “subtract method” (student language). Visually represent methods with arms at right angles, or one arm held diagonally.

•Main activity--solve nine Pythagoras problems with three problems at outdoor locations on grid (prior lesson); each provides a clue to an anagram. Provide three minute countdown--answers texted to teacher. Time--hands on head.

•Review learning.

•Announce winners of game and award prizes (dart board and random prize generator).

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