CHANCE AND DATA YEAR 7 UNIT PLAN	
Time Frame	3 weeks (14 lessons)
Context	Unit plan for two SHC classes (07MAT2 and 07MAT4)

Australian Curriculum Links:
Chance
•Construct sample spaces for single-step experiments with equally likely outcomes (ACMSP167)
•Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168)
Data representation and interpretation
•Identify and investigate issues involving numerical data collected from primary and secondary sources (ACMSP169)
•Construct and compare a range of data displays including stem-and-leaf plots and dot plots (ACMSP170)
•Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data (ACMSP171)
•Describe and interpret data displays using median, mean and range (ACMSP172)

Key Understandings:
•The organization of a set of data affects the analysis and interpretation of the data.
•Predictions can be made based on the analysis and interpretation of a set of data.
•The probability of an event is a number between 0 and 1 (and can be expressed as a percentage) that indicates the chance of something happening, and can be used to make predictions.
•Understanding of sample vs sample spaces, tree diagrams.

Essential Questions:
•How can we best present a certain set of data using a graphical representation?
•How can data be used to make predictions about real-world situations?
•How can all possible outcomes of simple experiments be represented?
•How can probability help us when critically thinking about data?

Outcomes:
•To develop technical ability to differentiate between types of data and optimal representation.
•To reinforce cognitive intuition of probability and chance.
•To understand concepts of mean (average), median, mode, and range.

Assessment:
Formative Assessment throughout. Formal worksheets. Final project.

Ongoing: Create a word wall of Chance and Data terms.

LESSON SEQUENCE:	**Resources:**
Lesson 1 & 2: Data collection and classification ***Source: Obook Year 7 Section 10A.*** **Objectives:** Hands-on data collection, organisation of data in a column and rows chart, identification of data types: •categorical--ordinal or nominal •numerical--discrete or continuous **Sequence:** Follow Obook section 10A	Obooks on each student's computer, math notebooks.
Lesson 3: Graphing introduction. **Objectives:** Understanding of many "right" ways to present and visualise data. **Sequence:** Review previous lesson key concepts (note: Lesson 3 and Lesson 4 are mostly diagnostic, and the order can be reversed). Presentation on data representation (PPT, 28 slides). Emphasise good elements of graph: Title, axes spacing and labels, keys. Lack of clutter, and clear visualisation of the data that is represented. Worksheet: Create stories for 3 graphs. Homework: Students create their data representation of a real or imaginary event or experiment which incorporates a relationship between the axes. Reinforce the creation of a graph that provides viewer with all necessary information: Title, Axes clearly marked (axes should be to scale), use full width or page(!), etc. Provide examples for x axes: time, calories eaten at lunch, amount of homework, travel distance, number of hours training for sport. Reinforce that the x axis generally used for the *independent variable,* and y axis is something that is measured based on the value of the independent variable, but for some data e.g. height vs. weight, it could be either. Assessment Criteria: neatness, creativity, all necessary information included with the graph, and explained with a written paragraph.	Presentation on iPad, web access for website view. Begin Word Wall. 30 copies of worksheet.

LESSON SEQUENCE:	**Resources:**
Lesson 4: Two coin flip, dot plots, and tree-diagrams. **Objectives:** To introduce quantification of probability with a simple sample space by plotting two-coin flip on a dot plot, and creating a tree diagram. **Sequence:** Begin by introducing a game--"we're gong to play a game--toss two coins at the same time--what is the probability of getting a head and a tail? For this game, we're going to organise into teams of three----two coin flippers and a recorder--and play the game three times with 10 coin flips per game, with each person taking a turn at being the recorder." Organise groups of three. Gather class, and explain how to record (dot plot). When three games are completed, create a neat dot plot with combined data from all three games for presentation. Hand out coins and let the chaos begin. After 15 minutes, quiet class and ask each group to show results. Ask why did we see a preference for heads-tails? Introduce concept of Sample Space--first look at tree-diagram and scaffold each branch with class. Note how the possible outcomes can also be presented in a table (First coin vs second coin). Ask: would the probability be any different if we tossed the same coin twice? Allow further experimentation using dot plots. Extra time: show probability ICT app on iPad. MAIN POINTS: Probability expressed as fraction, decimal, or percent. Probability for HT=0.5, probability for HH=0.25, probability for TT=0.25. All possibilities add up to 1.	Two coins per group, math notebooks.
Lesson 5: Plotting Bar Graph data based on Cyber-bullying article (Hobart Mercury, 2009)---Investigation involving visual representation of continuous numerical data. Extrapolating values on the graph. Relationship of axis (bivarate data), and trending of data.	

LESSON SEQUENCE:	**Resources:**
Lesson 6: Investigation involving visual representation of discrete numerical data. Mean (average), Median, Mode, Range concepts.	
Lesson 7: (More on creating and interpreting data displays.)	
Lesson 8: Probability with two dice. POE (Predict, Observe, Explain) Goals: Generate and define sample space Understand that intuition is often different from actual probability Generate, define, and discuss probability distribution Announce game-- predict whether odd numbers or even numbers are more likely for a roll of a pair of dice. Write numbers {2,4,6,8,10,12} and {3,5,7,9,11} on board. See who wants odd and even, and split class into pairs to play game. Many students might mistakenly believe that even has an advantage over odd, because there are more even numbers, {2,4,6,8,10,12} than there are odd {3,5,7,9,11}. Worksheet will be integrated into this lesson.	Dice (two per pair of students)
Lesson 9: Probability -- representation and prediction -- investigation with Lotto numbers.	
Lesson 10: Create a probability game (draft) Your task is to design a game which will make a profit for an upcoming fair. You can use a dice, a spinner, or anything else you can think of to play the game. You will need to think carefully about how to make sure the odds are in your favour. What's your game called? How much does it cost to play? What are the rules? What are the prizes? What is the probability of winning and losing? How much money do you expect to make if 100 people play? Create a chart of experimental and theoretical probability outcomes, e.g. frequency table. A criteria will be presented to students, i.e. how probable the game seemed to win versus the actual probability. PEER assessment.	
Lesson 11: Game time--create probability game for school carnival.	

Chance and Data Unit Plan for Meredith Balfe Year 7 classes. DRAFT by John Middendorf, PE3

Specific Schedule for Meredith's classes (M2=MAT2, M4=MAT4). Lesson Numbers.

Date				May3	May4
Day	6	7	8	9	10
P1					
P2					
P3					
P4				M2 1	
P5					
P6					M2 2

Date	May7	May8	May9	May10	May11
Day	1	2	3	4	5
P1				M4 5	M2 7
P2					
P3	M2 3		M4 4	M2 6	M4 6
P4	M2 4		M2 5		
P5					
P6		M4 3			

Date	May14	May15	May16	May17	May18
Day	6	7	8	9	10
P1			M4 7		
P2				M4 9	
P3					
P4				M2 8	
P5			M4 8		
P6					M2 9

Date	May21	May22	May23	May24	May25
Day	1	2	3	4	5
P1				M4 12	M2 14
P2					
P3	M2 10		M4 11	M2 13	M4 13
P4	M2 11		M2 12		
P5					
P6		M4 10			

Rationale on misconceptions--by John Middendorf for EMT620 at UTAS

In contrast to mistakes, a misconception in mathematical reasoning can occur when a learner, in constructing mathematical knowledge, extends a mathematical concept or "rule" in an fallacious manner. With modern constructivist teaching pedagogy, which encourages the learner to build on innate knowledge, the obligation for the teacher to be vigilant in identifying misconceptions becomes especially critical. The primary pedagogy involves a diagnosis focused on providing students opportunities to comprehensively explain their logic. Once a misconception is identified, the teacher then provides learning opportunities and experiences that encourages self-recognition of the misconception, which then prompts the student to modify their cognitive schema by accommodating the revised conception. Built on the Piagetian concept of disequilibrium, in order for a learner to abandon a misconception and accommodate conceptual exchange, there needs to be dissatisfaction with the misconception (Hewson, 1992). Note that the misconception diagnostically informs the level of student understanding; as Osborne expressed, "We must start where the child is" (1982).

The misconception I have chosen to address relates to the "law of small numbers" (Tversky & Kahneman, 1971, p. 105) which is the misconception that a small sample represents a larger group. In contrast, the "law of large numbers" reflects the accurate inference that the larger the sample, the larger the probability of the sample representing the whole. The misconception can arise from an intuitive extension of the quantitative part-whole concept of numbers, and is interrelated to other misconceptions; Fischbein and Schnarch (1997) outline seven common probabilistic misconceptions:

1. Representativeness (i.e., the overestimation of an event's likelihood due to its similarity to the parent form).
2. Negative and positive recency (i.e., coin tosses have "memory").
3. Compound events (i.e., similar outcomes from a two-coin toss are not differentiated).
4. Conjunction fallacy (i.e., personal bias influences perception of likelihood)
5. Sample size (i.e., variation from mean is misjudged in smaller sample).
6. Availability (i.e., the likelihood of more 'available' events, such as a isolated event portrayed in the media, is overestimated).
7. Falk Phenomenon (i.e., the effect of a gap in time on probability).

The "law of small numbers" is related to the misconception of representativeness (Fischbein & Schnarch, 1997) and also linked to the misconception of recency, as the notion that a random generated set can have "memory" is similar to the expectation that consistent patterns will be seen in small samples.

I chose this difficult misconception because it highlights the importance of reinforcing mathematical language and informal mathematical reasoning as the groundwork for statistical numeracy, as the formal analysis of probability requires abstract combinatorial logic[1], and as Confrey (1990) argues, preferentially initially presented with a subjective approach to reinforce intuition. Probabilistic misconceptions can detrimentally affect critical thinking skills when an individual is presented with data and sampled information. The misconception can be diagnosed[2] with the "hospital problem":

> In a small town at a small hospital, 10 babies are born per day on average. In the nearby city at a large hospital, 50 babies are born per day on average. Each hospital has a celebration when 60% of the births on a given day are girls; they call this a "girl day". Which hospital--the small one or the large one--is more likely to have a girl day, or would they both be equally likely to have a girl day? (adapted from Fischbein & Schnarch, 1997)[3].

Most children have the misconception that both hospitals have equal likelihood for girl days, with the larger hospital being the second choice (Fischbein & Schnarch, 1997)[4].

Diagnostic assessment continues with an understanding of the level of development. Beginning with Piaget and Inhelder in 1951 (Davies, 1965), researchers have linked probabilistic reasoning with cognitive development stages, yet no complete theory has emerged (Way, 2003). Way distinguishes three stages:

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[1] Because of its dependence on abstract reasoning, the formal analysis of probability would be more appropriate when learners have reached the formal operational stage.

[2] This problem would only be appropriate for the upper-primary grades, but a similar problem could be created that was more contextual for younger students. For example: *Ann has a box of chocolates, filled with jelly-filled chocolates and caramel-filled chocolates, which all look the same, but there are less caramel-filled chocolates than jelly-filled ones. Bob picks two chocolates and hopes that both are caramel, while Cara picks six chocolates and hopes that all six will be caramel. Do you think Bob or Cara will be more likely to have their hopes realised, or are they both equally likely? Explain why.*

[3] The original "hospital problem" was posed by Kahneman and Tversky in 1972.

[4] The smaller hospital, of course, is the correct answer, due to the smaller sample size.

Stage 1: *Non-probabilistic thinking.* Students in this stage have minimal understanding of randomness, are reliant on visual comparison, and are unable to order likelihood.

Stage 2: *Emergent Probabilistic Thinking.* Students in this stage can recognise impossible or equally likely sample spaces, and use additive proportional reasoning to make generalisations about likelihood.

Stage 3: *Quantification of Probability.* Students in this stage begin to recognise the relationship between randomness and likelihood, use multiplicative proportional reasoning, and are developing a language of probability (e.g. correct use of "chance" and "more likely") (Way, 2003).

Addressing the misconception is dependent on the diagnostic assessment and involves exposure to activities which reinforce statistical numeracy intuition[5], scaffolded with the development of probabilistic language. A three tier approach is recommended:

1. Basic understanding of statistical terminology.
2. An understanding of statistical language and concepts when they are embedded in the context of a wider social discussion.
3. A questioning attitude one can assume when applying more sophisticated concepts to contradict claims made without proper statistical foundation (Watson and Moritz, 2000, p. 45).

In particular, Watson and Moritz focus on developing the technical understanding of the language of sampling though a series of explorations that differentiate the common usage of "sampling" (in reference to a homogeneous whole such as a "sample of cheese" or a "sample of blood") with the statistical usage through explorations of group variability, such as the height or weight of students, which highlight the relative representativeness of larger samples.

Probability lesson plans are informed by the "big ideas" already discussed, and "simulation is a technique used for answering real-world questions ... in which an element of chance is involved" (Van de Walle, Karp and Bay-Williams, 2010, p. 456). POE (predict-observe-explain) investigations with ICT manipulatives (see NLVM, 2012) are appropriate. Using contextual examples drawn from student's interests assists with engagement. Parallel lessons in equiprobability would also be of benefit in addressing sampling misconceptions. In conclusion, appropriate

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[5] Intuition in this context is the secondary intuition developed by systematic instruction that are “acquisitions that have all the characteristics of [primary] intuitions but . . . are formed by scientific education, mainly in school” (Fischbein, 1975 as cited in Greer, 2001, p. 18).

probability pedagogy at each stage of a child's development is important for the progressive construction of statistical numeracy, providing an increased intuition of situations where statistical bias can occur, as well as "motivate the questioning attitudes required of future citizens" (Watson & Moritz, 2000).