LESSON PLAN--Addition with the Empty Number Line (ENL)

Year Level: Year 2

Lesson Duration: 45 minutes.

Curriculum Links: Number and Algebra, Year 2: "Understanding includes ... partitioning and combining numbers flexibly" and "perform simple addition and subtraction problems using a range of efficient mental and written strategies" (ACARA, 2012).

Objectives/Outcomes:

- Consolidate and extend mental partitioning of a two-digit number.
- Use the empty number line (ENL) as a reliable algorithm to add two-digit numbers.
- Consider and develop multiple strategies for addition problems.

Assessment: Formative assessment throughout, and informal assessment at end assessing learning of the lesson and informing the degree of review and/or conceptual reinforcement required prior to further ENL lessons.

Prior Knowledge:

- Mental arithmetic of sums to 20, bridging through 10 (e.g. 8+6=14).
- Addition bonds of 10 (e.g. 7+3, 8+2, etc.).
- Basic number line concepts.

Previous Lesson: Addition using a 100-bead string (with 10-structure).

Subsequent Lessons:

- Empty number line--subtraction.
- Mental Arithmetic (without the ENL) involving partitioning of both two-digit numbers in addition problems (1010 strategy).

Groupings: Normal group seating arrangements (4-5 students per table, 5 tables total). There will be whole class discussions, and Think-Pair-Share group work at tables. Equipment: Math books, cord and clothespins.

LESSON INTRODUCTION (teacher spoken contributions in italics)

<u>Part 1 (5 minutes)</u>

What?: Start with simple jump counting exercise.

Why?: Prepare for maths with simple task and clear instructions. Activate prior knowledge of addition and patterns by jump counting by 10's.

"Take out your math-books, and make three columns on a new page. Put 7 at the top of the first column."

Provide few moments and settle class.

"Now add 10 to the 7, and write the sum below the 7. Continue jump counting by 10's down the column."

Repeat for next two columns (use 4, 29). Walk room, ensure everyone is prepared, understands task, and can jump count up to 100. Acknowledge students who bridge 100.

<u>Part 2 (6 minutes)</u>

What?: "Guess The Number" game.

Why?: Review relative locations along a number line, activate memory skills.

Props: Cord stretching across the room in front of the class, red and green clothespins.

Settle class and focus attention.

"We are going to be looking at number lines today and the relative values of numbers. This cord represents a number line between 0 and 100. I am thinking of a number between 0 and 100--can anyone guess my number?" (e.g. 42).

Explain colour coding. For each guess, ask approximately where number would be on number line, place either the green (for too low) or red (for too high) clothespin on line at appropriate location, narrowing the range with each guess. If guess is outside range (e.g. 64 when green and red markers represent 20 and 55), scaffold by reminding of numbers each clothespin represents, e.g.:

"This red clothespin represents 55. Is 64 higher or lower than 55? So where would 64 be? [noting location]. Can anyone provide another guess between 20 [pointing to green marker] and 55 [pointing to red marker]?"

Continue game. For the student who correctly guesses, invite student to front to pick a number and play game with class (student whispers number to teacher so teacher can moderate the game).

ICT: Number line estimation can be further reinforced with an interactive game (see BrainPOP, 2012).

MAIN LESSON

Part 1: Whole Class (12 minutes)

What?: Introduce empty number line for addition problems.

Why? Encourage multiple and efficient strategies of mental computation by activating and extending mental partitioning. Compensation, complementary addition, counting on (when appropriate) also considered. Addition commutativity and bridging through 10 reviewed.

Describe the ENL as a number line without markings (draw one on board), explain relative scaling (arbitrarily mark a number, then draw a series of scaled 10's and 1's). Ask what is different about this number line, and scaffold responses appropriately. Next, write "36+57=?" on the board above a new empty number line.

"How can we solve this addition problem using an empty number line?"

Pause, then demonstrate various strategies of adding 36 and 57 on ENL, using the following solutions and ensuring consistent scaling of the jumps:

36+(5x10)+7 (noting conversion of 50 into five 10's) [1]

36+50+7 (ask if anyone can mentally add 36 + 50 prior to adding the 7)

Explicitly add the sum of the jumps to check each solution. At this point, remind students of additive commutativity, e.g.:

"When we add two numbers, is the order important? For example, 2+5 is 7, and so is 5+2, correct? So can we start with 57 on the empty number line, and add 36?"

Continue with examples starting with 57:

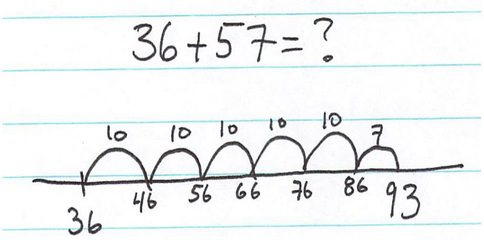
57+(3x10)+6

57+30+6

57+30+3+3 (noting that we can split the 6 into 3+3)

57+3+30+3

For each solution, explicitly write each jump value and jump location, as shown in Figure 1. Verify summation of jump values as one of the addends.



[1] Notation: 23 + (5 x 10) + 8 represents starting at 23, jump five 10's, then bridging to 81 by adding 8.
23+50+8 represents starting at 23, jumping 50 to 73, then bridging to 81.
23+ (5 x 10) + 7 +1 represents starting with 23, jump five 10's, jump 7, jump 1. Etc.

Figure 1: Explicit notation for each jump (example shows conceptual bridging from 86 to 93)

Erase the board and write "48+37=?" above an ENL. Ask students to suggest various methods, and have several students demonstrate on a new empty number line on the front board. Using student language, verbally reinforce and label each strategy utilised, e.g.:

> *"Sam is jumping three 10's to 78, then adding 2 to get to the 'nice number'[2] of 80, then adding 5 to arrive at 85".*

Scaffold strategic awareness by categorising techniques. When "counting on" strategies are used, ask if there are solutions using fewer jumps. Ensure N10 solutions (jumps by 10 and jumps by a multiple of 10) and A10 strategies are demonstrated (see rationale), and reiterate strategies if not student discovered. For errors, ask questions that focus on the sum of the jumps and ensure proper notation (Figure 1).

<u>Part 2: Think Pair Share work (15 minutes)</u>

What? Independent work using the ENL followed by seeing other's solutions.

Why? Encourage social construction of alternate and efficient solutions by considering various strategies of addition problems.

Transition class to table work, ask students to show their work using ENL's on a new page in their maths books for the following:

18+14

24+58

38+37

54+29

Provide few minutes for independent quiet work, wander the room and assess, assisting struggling students while challenging advanced students to discover alternative methods. Ask students to explicitly explain steps and scaffold using student language, e.g.:

> *"Can you tell me about this big jump to 74?"*

For student responses, elicit verbalisation of both jump size and resulting number line value.

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[2] The term "nice number" to reference a multiple of 10 is used in Van de Walle, Karp & Bay-Williams (2010), p. 221.

Next, in groups of 2 or 3 (two groups per table), students compare answers and note whether other students did problems similarly or differently. Wander room, provide five minutes, then ask whole class to show hands if they solved each addition the same or differently than their partners, and whether they thought their solution, or their partners, was simpler. Request detailed responses from at least one student at each table. Survey solutions for the final problem to see if anyone used compensation methods for the 54 plus 29 (54+30-1); if so, alert class to the new method and have student demonstrate on front board--otherwise, outline compensation solution.

CONCLUSION AND ASSESSMENT (7 minutes)

What? Teacher directed discussion of various methods that benefit mental computation.

Why? Consolidate learning and to assess understanding of empty number line.

Ask,

"Does anyone find the empty number line a reliable way to add two numbers?"

"Do you have other methods you prefer to add two numbers?"

Discuss responses.

Pose new question as a verbal context problem that students can either solve in their heads or use their preferred addition method:

"If I bought two books, one priced at $25 and one at $48, how much is the total?"

Strategic pause, then:

"Hands up if you think you can tell me the answer?"

"Was anyone able to solve it in their heads?"

"Did you initially consider the various ways it could be solved?"

Highlight various mental strategies offered. Next, questions for "hands up" responses:

"Who started with $25, then added the $48? What did you do next?"

"Who started with $48, then added the $25? What did you do next?"

Explicitly acknowledge efficient strategies using N10, N10C, and A10 methods. Review questions, inform next lesson will be ENL for subtraction problems. Compile observational notes on struggling/advanced individuals, and for the class as a whole.

ICT Resources: For students who need further reinforcement, use "Number Line Arithmetic" (National Library of Virtual Manipulatives [NLVM], 2012a). Advanced students can extend with "Number Line Bounce", a puzzle game based on number line jumps (NLVM, 2012b).

RATIONALE

This lesson introducing the empty number line (ENL) is sequenced to follow an addition lesson using a structured bead string[3] (Klein & Beishuizen, 1998), and to be succeeded by a subtraction lesson using the ENL. This lesson is designed to encourage multiple and efficient strategies of addition computation which enable "a reduction of the memory load while solving a problem (Baroody, 1987 as cited in Klein & Beishuizen, 1998 p. 447).

Three partitioning[4] methods are emphasised in this lesson (Klein & Beishuizen, 1998):

1. N10: partitioning an addend into tens and ones to enable simpler mental additive "jumps".
2. N10C: a variant of N10 often used with numbers ending in 8 or 9, where a multiple of 10 jump is followed by a compensating subtraction (e.g. 29+54=(54+30)-1).
3. A10: a variant of N10 where the ones are also partitioned to allow a bridging jump to a multiple of 10 (e.g. 57+34= (57+3)+30+1).

The lesson also encourages considering re-ordering addends to simplify subsequent computation. Additional methods of mental computation, such as doubles, combinations (e.g. 65+37=100+2, recognising 65+35=100), or 1010 strategies (where both numbers are partitioned) are not emphasised, but the teacher should not discourage individual utilisation during this lesson,[5] as "students will often construct knowledge in unplanned ways" (Muir, 2008, p. 362).

The ENL, "a very powerful model for the learning of addition and subtraction up to 100" (Klein & Beishuizen, 1998, p. 443), is the scaffold to extend number concepts and

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[3] The structured bead string has 100 beads structured by 10's: 10 red beads, 10 blue beads, etc.

[4] Note: terminology differs in the literature. Thompson (2007) calls the N10 process "sequencing", though he acknowledges other literature that identifies it as partitioning (Thompson considers only the 1010 as partitioning). Partitioning in this paper refers to the general method of "break(ing) up numbers in a variety of ways" (Van de Walle, Karp & Bay-Williams, 2010, p. 227), and used in two conceptual senses: e.g. the partitioning of 56 into 50 and 6, which uses base 10 logic, and partitioning of a single digit number as an addition bond to a multiple of 10.

[5] In particular, 1010 strategies cannot be demonstrated with an empty number line and in fact are based on conceptually different procedures, thus separate and distinct lessons involving double partitioning strategies are recommended. (Thompson, 2007).

mental computation strategies. "With support of the ENL model this sequential strategy N10 becomes much easier to acquire and use, and turns out to be a very flexible and effective mental computation procedure." (Beishuizen & Anghileri, 1998, p. 520). Other benefits to using ENL algorithms are the explicit and visual error checking, which assists understanding: "if children were able to invent their own methods and explanations that showed up in patterns of error, they would also be building their own understanding of appropriate ways of thinking rather than simply taking in a teacher's explanations" (Booker, 2010, p. 26). The design of the lesson is for students to develop "their own strategies by exploring, discussing, and justifying their thinking and solutions (Heirdsfield, 2011, p. 96).

The teacher's guiding role during the entire lesson should be focused on considering the paths to student solutions both in terms of efficient number of steps and the mental memory load. The mental memory load is something the teacher will be able to discern affectively when the student is expressing the reasoning behind each computation; for example, if the degree of partitioning creates errors due to too great a conceptual step for a particular student, the solution can be reliably adjusted to include more steps but require less mental memory load (or vice versa). The teacher is able to guide this process with Socratic scaffolding with questioning specific to individual steps in the process.

Differentiation is especially critical in a lesson involving mental computations, as some studies suggest there "can be as much as a 7-year difference in the range of understanding at age 11 of certain concepts" (Thompson, 2000, p. 292); in this lesson, the four individual work problems are structured in order on conceptual challenge. The lesson encourages a reduction in reliance of "counting on" strategies, and by suggesting conceptual examples that are within the student's zone of proximal development (ZPD) (for example, simpler partitioning such as 14=10+4 can be provided), the concepts can then be graphically extended on the ENL. The teacher can scaffold advanced students on the spot with individual work extending the concepts with more challenging word problems.

The lesson is constructively aligned with whole-group instruction followed by smaller Think-Pair-Share groups, enabling students to "compare their strategies with others' strategies, and critique the strategies" (Heirdsfield, 2011, p. 100) and encouraging active construction of knowledge from both individual and social cognitive reasoning. The lesson conclusion brings the whole class back together to review and highlight the key strategies used to efficiently compute the additions.

The teacher assists the student's creation of structured schema of mental strategies by first modelling the procedures and then providing free reign on chosen strategies; thus the teacher needs to be poised with a broad awareness of mental computation methods in order to respond to "teachable moments" (Muir, 2008, p. 362) by providing appropriate responses that enhance conceptual understanding. Throughout the lesson, it is important for the teacher to ask students to explain their reasoning of each step to identify the strategy and expose any incongruences in reasoning. Primarily, the lesson is designed to encourage higher level thinking strategies by providing students with a visual tool to scaffold the conceptual steps to improved mental computation methods.

REFERENCES

Australian Curriculum Assessment and Reporting Authority. (2012). The Australian curriculum v. 3.0: Mathematics foundation to year 10 curriculum. Retrieved from: http://www.australiancurriculum.edu.au/Mathematics/Curriculum/F-10

Booker. (2010). Teaching primary mathematics. In T. Muir (Ed.), Primary and early childhood mathematics pedagogy (pp. 15-54). NSW: Pearson.

BrainPOP. (2012). Battleship numberline. ICT resource available at: http://www.brainpop.com/games/battleshipnumberline/

Klein, A.S. & Beishuizen, M. (1998). The empty number line in Dutch second grades: Realistic verses gradual program design. Journal for Research in Mathematics Education, 29 (4), pp. 443-464.

Muit, T. (2008). "Zero is not a number: Teachable moments and their role in effective teaching of numeracy. Proceedings of the 31st Annual Conference of the Mathematical Education Research Group of Australasia, pp. 361-367.

Mutchmor, M. (2012). Using the empty numberline as a mental teaching strategy. From: www.uwosh.edu/coehs/mindsongmath/concepts/documentsEmpty_Number_Line.pdf

National Library of Virtual Manipulatives. (2012). Number Line Arithmetic. Retrieved from http://nlvm.usu.edu/en/nav/frames_asid_156_g_1_t_1.html

National Library of Virtual Manipulatives. (2012). Number Line Bounce. Retrieved from http://nlvm.usu.edu/en/nav/frames_asid_107_g_2_t_1.html

Thompson, I. (2007). Deconstructing calculation part 1: Addition. Mathematics Teaching, 202, pp. 14-15.

Thompson, I. (2000). Teaching place value in the UK: Time for a reappraisal? Educational Review 52(3), pp. 291-298.

Van de Walle, J.A., Karp, K.S. & Bay-Williams, J.M. (2010). Elementary and middle school mathematics: Teaching developmentally (7th ed.). Boston: Pearson.