Tasmanian Camels Problem

Solution 1 (my solution):

My first attempt to solve the Tasmanian Camels problem (Appendix A) was to solve it in my head, as I began trying the problem while driving down to the Tasman Peninsula on a family camping trip. I began with creating a suitable mental notation and tried three variants:

1. Camels labelled A1, A2, A3, A4 and B1, B2, B3, B4
2. Camels labelled -4, -3, -2, -1, and 1, 2, 3, 4
3. Camels generally labelled A and B, and number line labelled 1-9.

The third mental notation above was the most successful mentally. I was able to make a few moves mentally, but would generally lose track after 3 or 4 moves. This strategy required remembering a sequence of nine characters after each move, which typically was a sequence similar to "AAAB-blank-ABBB".

Gathercone & Alloway (2007) define the concept of Working Memory (WM) as "our ability to hold in mind and mentally manipulate information over short periods of time (as cited in Ginns, 2010). Furthermore, Miller (1956) states that we can "rehearse 7+/- 2 'chunks' of information" (as cited in Ginns, 2010). My strategy of recalling nine chunks at a time, for repeated sequences, taxed my working memory, and along with the distractions of family chatting in the car, I was unable to solve the Tasmanian Camel problem mentally.

Once we got to our camp on the beach, I set upon solving the Tasmanian Camel problem with visual aids. I created a number line in the sand, and used dark rocks and white shells to represent the camels.

It quickly became clear that the problem would have symmetrical solutions based on the first move; thereafter, the first move became clear: I moved the black rock to the empty space at spot 5. The second move was clear: to jump the white shell to spot 4; the alternative, moving the black rock from spot 3 to spot 4 led to an obvious impasse. The "wrong" moves were key to my ability to later solve the problem.

I came to the sequence illustrated in Figure 1.

Figure 1: after first two moves.

At this point there are two choices:

1. Move the black rock at spot 5 to spot 6.
2. Move the white shell at spot 7 to spot 6.

The first choice (#1) led to a forced jump of either the white shell at spot 7 to spot 5, or the black rock at spot 3 to spot 5. Both of these resulted in a "double camel" which became apparent would make the problem impossible to solve further.

At this point, I developed my initial "rule" namely, to prevent the "double camel" formation after camels have moved from their original position. It also became clear that to accomplish this, I would need to keep the black rocks on odd numbered spots, and thus I revised my rule. With my new rule in mind ("black rocks on odd-numbered spots"), I sought moves that would put the black rocks on the odd numbered spots on the number line.

Subsequent moves are shown in Figures 2 and 3.

Figure 2: After move 4--black rocks seeking odd spots.

Figure 3: After move 5.

After move 5, I realized that one of the aspects that made this problem challenging. Although the correct sequence for the first two moves is to move a camel from one side, and then the other, this pattern quickly diverges into a different pattern. The five moves leading up to Figure 3 have the following pattern of moves: black, white, white, black, black. One seeking symmetry intuitively might be tempted to move a white shell at this point, but the correct solution is to move a black rock again (from spot 2 to spot 3), leading to Figure 4.

Although moving a white shell after Figure 3 led to a conflict of my rule, I did initially proceed incorrectly at this point, perhaps due also to my lack of faith in my rule, as my rule was yet unproven, having not led to a successful completion of the problem at this point.

Figure 4: Essentially solved after step 6.

Returning to my rule, I came to the sequence illustrated in Figure 4. Here, the problem is essentially solved, as it becomes apparent that the white shells can all escape to the left by leapfrogging the black rocks, after which the black rocks can continue unimpeded on their way. Overall, it took me about 15 minutes to solve the problem.

Solution 2 (John B's solution):

John B. solved the Tasmanian Camel problem differently. He immediately asked for a pen and paper, which I supplied. Initially there were several questions not relevant to the solution, including:

"Can the camels move 1/2 spaces?"

"Are the camels all the same size?"

He also asked questions relevant to the problem:

"Can camels jump over more than one camel?"

"Is it possible to solve this puzzle?"

"Is four (camels) a significant number?"

"Would it be easier to solve with an odd number of camels—3 or 5?"

As a interested observer, I only took notes and did not answer the questions directly. As John proceeded with the problem, using pen and paper, he quickly arrived at a suitable written notation, namely a right arrow (>) for the camels starting on the left side, and a left arrow (<) for camels starting on the right side.

John solved the problem in approximately 20 minutes using pen and paper. His notes are shown in Figure 5.

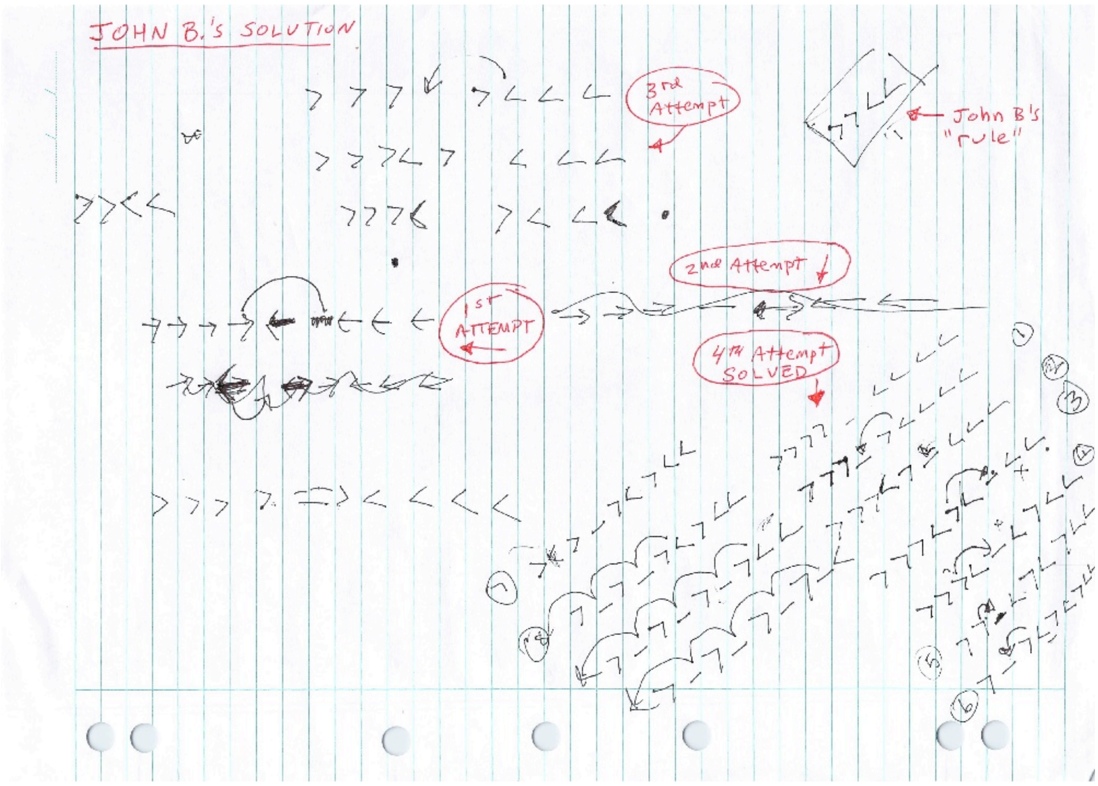

Figure 5: John B's solution.

After three unsuccessful attempts, John also developed a rule to solve the problem. He drew a picture of the situation to avoid: two camels together moving right, and two camels together moving left (“>><<”). He first expressed this rule as “As soon as you have two facing the same direction, you’re stuffed”. He then wrote his rule on the top right corner of Figure 5, and for each stage of his solution, he would look at the options for each move, and decide that the only possible “next step” was the one that led to a sequence that did not visually have a sequence with “>><<” in it. He was able to see the wrong move in his head from looking at his previous step, and after marking the alternative move with a arrow, he was able to write down the next step strictly by looking at his notation.

Throughout the solving of the problem, John was interested in the symmetry of the problem. He also considered that the solution might involve camels from the same side jumping over one another (“bugger, that’s more possibilities”, he realized), but decided to try to work out a solution not utilizing that option. Once he decided on his rules, he methodically proceeded step-by-step to the correct solution.

After he solved it, John asked how I solved it, and when I told him I solved it using objects, he stated that he hadn’t thought of using objects; interestingly, I thought it would be much harder to solve just using pen and paper until I witnessed his solution.

We also discussed the minimum number of steps required to solve the problem, and realized that we had both solved it in minimal steps, as each “branch” of the problem only led to one correct subsequent step. Another interesting difference of our respective solutions was that he was not sure he had solved it until step 8, which appeared on his notes as “>_>_>_><<”; on the other hand, I could see the problem as essential solved by step 6 (Figure 4). Though his approach was much more methodical, the written notation made it a bit more difficult to see the final steps that led to the complete passing of all camels.

Reflection

The Tasmanian Camel solution requires spatial reasoning in problem solving. Problem solving is “ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively” (Shape of the Australian Curriculum: Mathematics, 2009, p.6). Furthermore, visualization of the problem is essential; “Visualization refers to the understanding of the problem with the construction and/or the use of a diagram or a picture to help obtain a solution” (Bishop, 2009, as cited in Deliyianni, E., Monoyiou, A., Elia, I., Georgiou, C. and Zannettou, E., 2009).

Both John B. and I solved the problem with visual aids: a diagram in John B’s solution, and a physical representation of objects in my solution. The conceptual leap to phrase the problem in visual terms is the first step. I had asked others to solve the Tasmanian Camel problem, and some were unable to conceive of creating a visual representation. Others, who did think of creating visual aids but were still unable to solve it, failed due to confusing nomenclature of their visual aids. For example, my nephew tried to solve it with eight different coloured drawing pencils, and he quickly became confused which camels were originally from each side. Both John B. and I chose visual aids that had a clear nomenclature.

Creating alternate ways of phrasing a given problem is an example of cognitive pluralism, defined as "varying sense modalities, to emphasise the multiple modes and practices that are available to generate, communicate, learn and display knowledge" (John-Steiner, 1995, as cited in McLoughlin, 1997). As a maths teacher, helping students expand their cognitive pluralism will be of benefit in order to enhance problem solving and critical thinking skills.

Even with an appropriate visual representation, another cognitive step is essential for an efficient solution; namely, the use of a heuristic. Both John B. and I created "rules" to solve the problem. In John's case it was a situation to avoid, and mine evolved into a situation to seek (namely, put the black rocks on odd spaces).

Martinez (1998, p. 605-606) defines a heuristic as "a rule of thumb. It is a strategy that is powerful and general, but not absolutely guaranteed to work... All heuristics help break down a problem into pieces." Furthermore, Martinez states that in the process of searching for a heuristic, mistakes are a natural process: "If no mistakes are made, then certainly no problem solving is taking place" (p. 608). Understanding that mistakes are a prerequisite in finding a heuristic, and that a heuristic is a benefit in problem solving ability, it follows that we should celebrate mistakes as the natural process to finding a suitable heuristic for a given problem.

The Tasmanian Camel problem, according to game theory is a problem with "perfect information" (Pauly, 2002) according to game theory. Perfect information problems can be solved algorithmically, with all possible branches at each node of the problem explored to its finality; but this would result in an inelegant solution. To successfully solve the Tasmanian Camel problem, an appropriate visual representation, and a suitable heuristic became the keys to an efficient solution.

References

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Appendix A—Tasmanian Camels Problem

Four Tasmanian camels travelling on a very narrow ledge encounter four Tasmanian camels coming the other way. As everyone knows, Tasmanian camels never go backwards, especially when on a precarious ledge. The camels will climb over each other, but only if there is a camel sized space on the other side.

The camels didn't see each other until there was only exactly one camel's width between the two groups. How can all camels pass, allowing both groups to go on their way, without any camel reversing?