Form Finding for Anticlastic Membrane Stuctures

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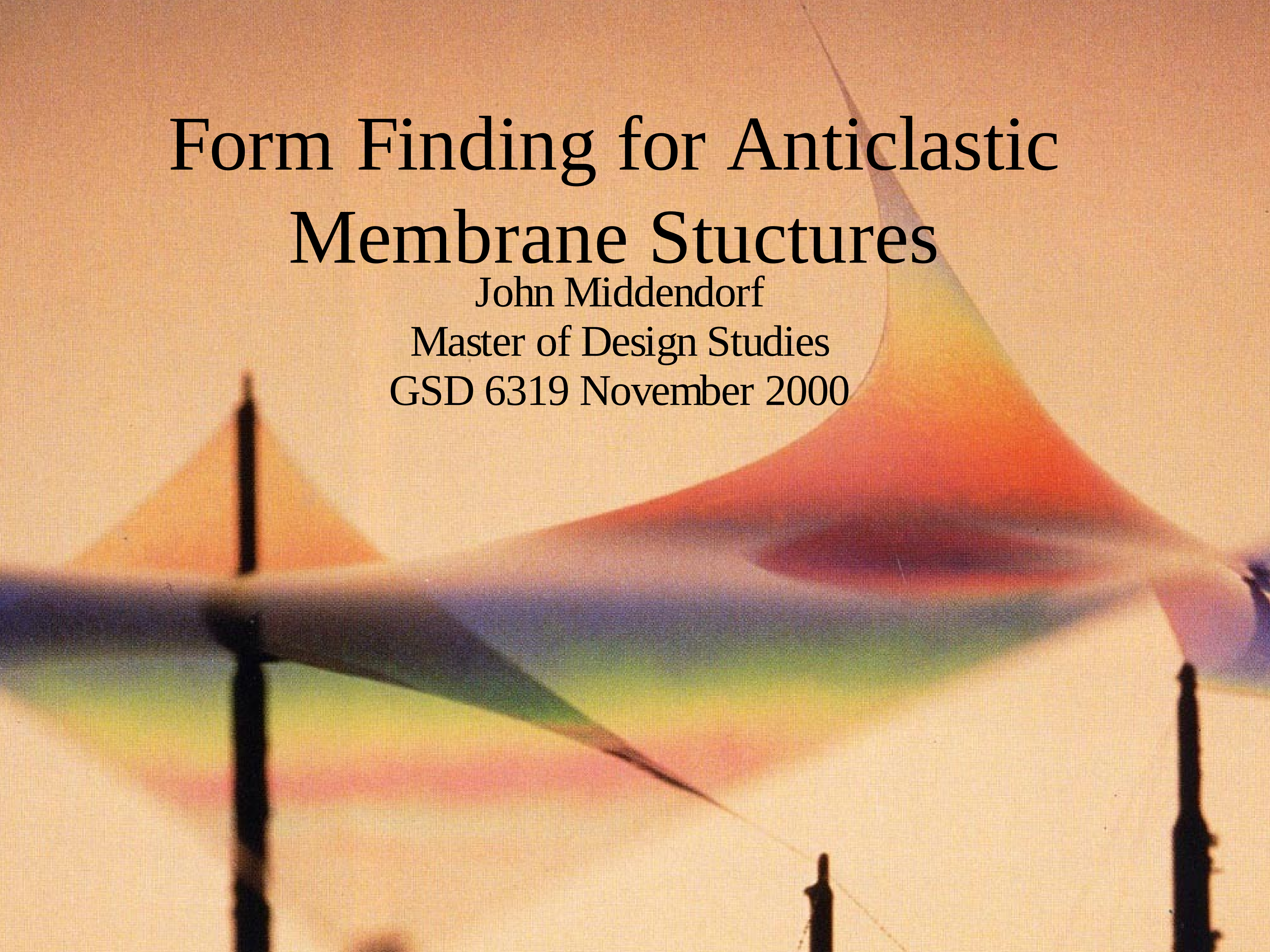
Master of Design Studies
GSD 6319 November 2000



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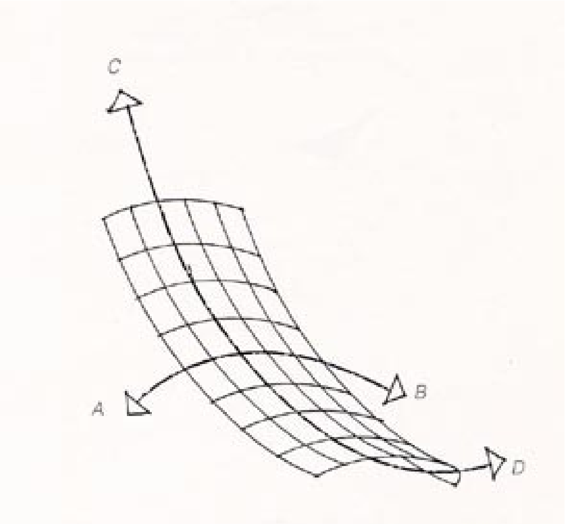
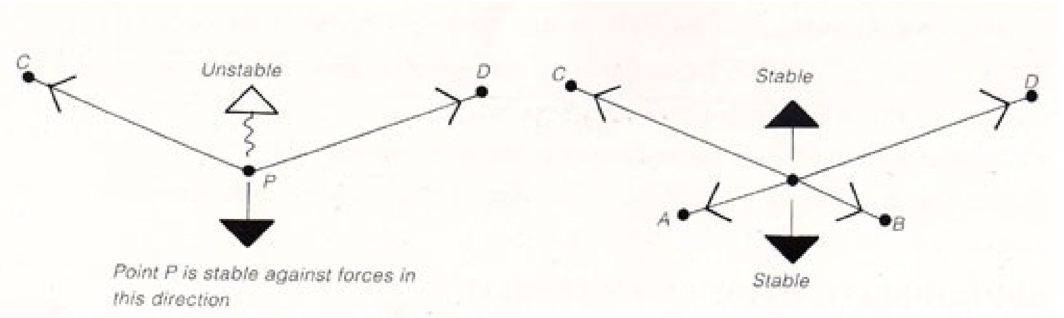
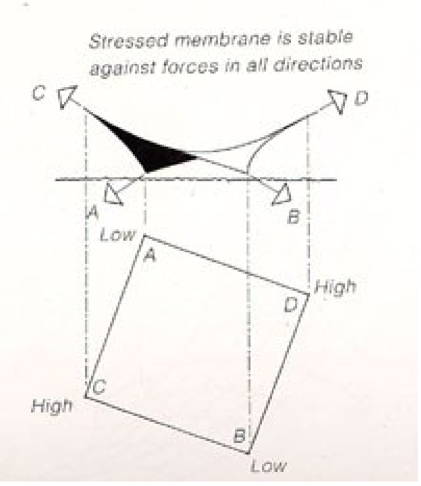
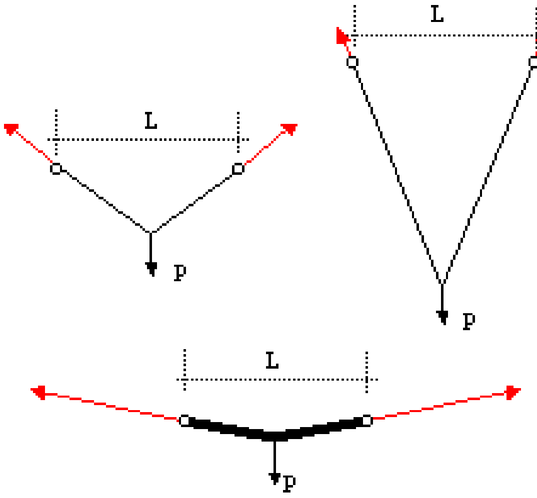
Double Curvature



17. The doubly-curved saddle-shape typical of stressed fabric structures

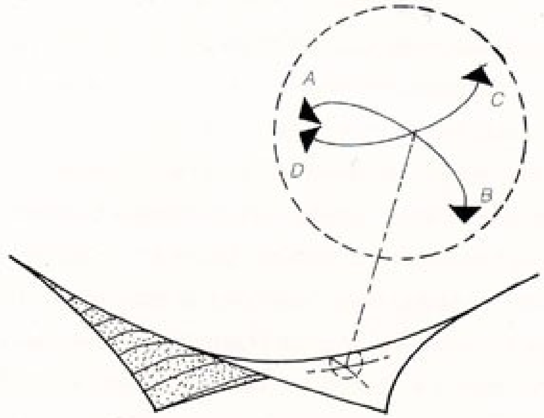




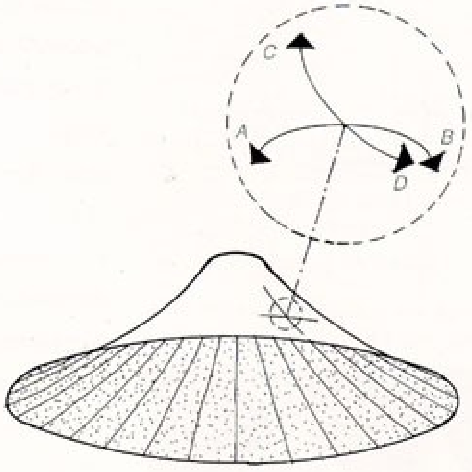


Large radius of curvature results in large forces.

Anticlastic Shapes



Hyperbolic Paraboloid

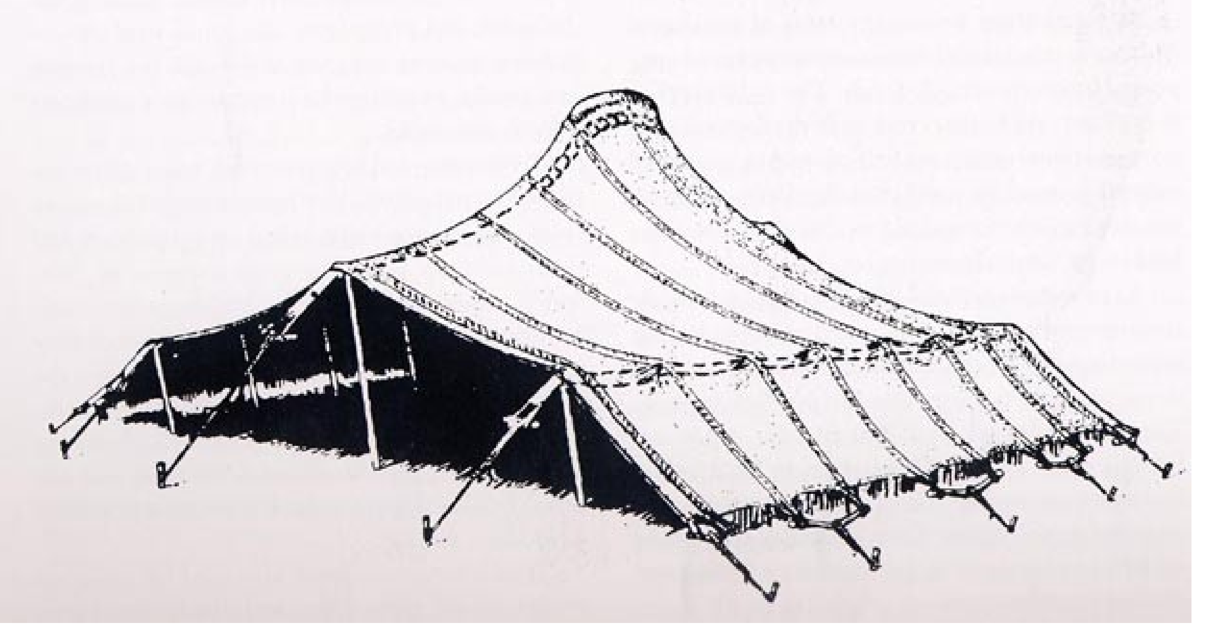


Double Ring Cone

Valley and Ridge

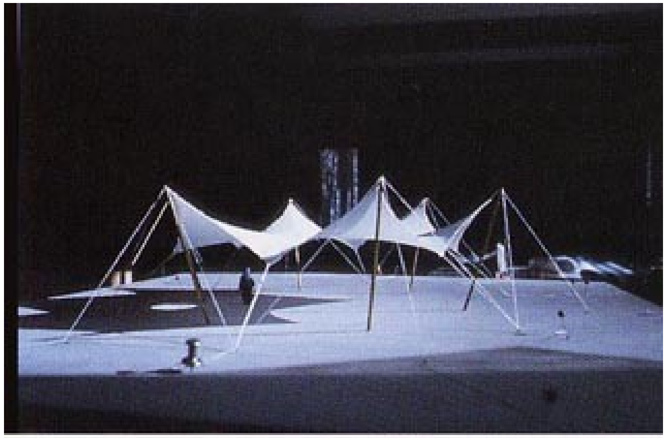
Arch Support

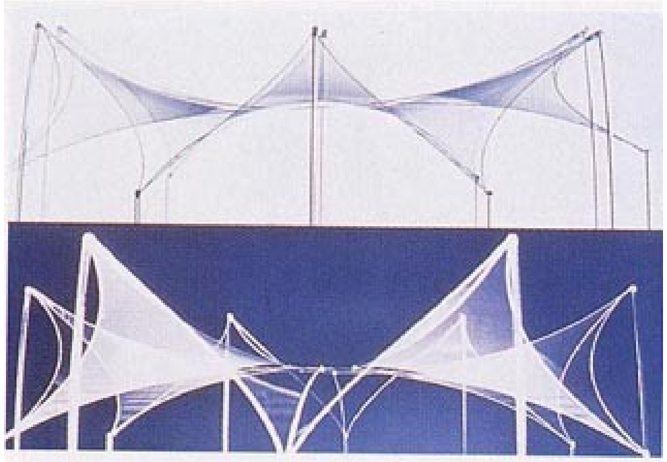
Development of Anticlastic Fabric Structures



Bedouin Black Tent

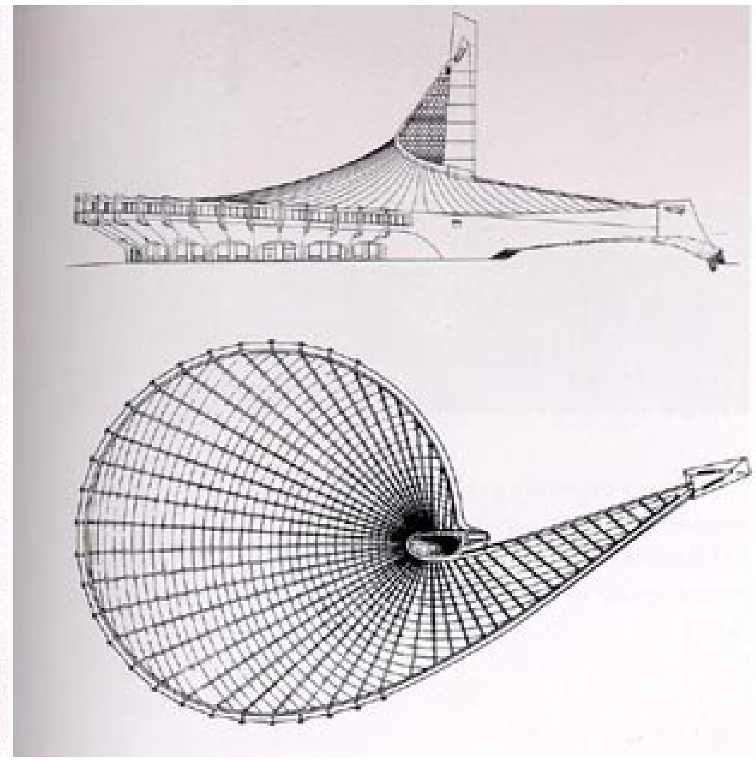
Dance Pavilion, Federal Garden Pavilion, 1957





Frei Otto

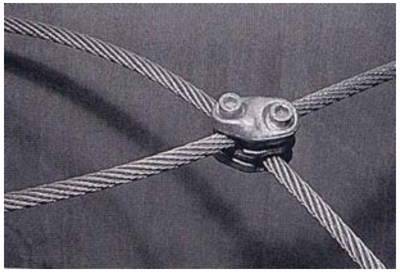
Olympic Stadiums, Tokyo 1964

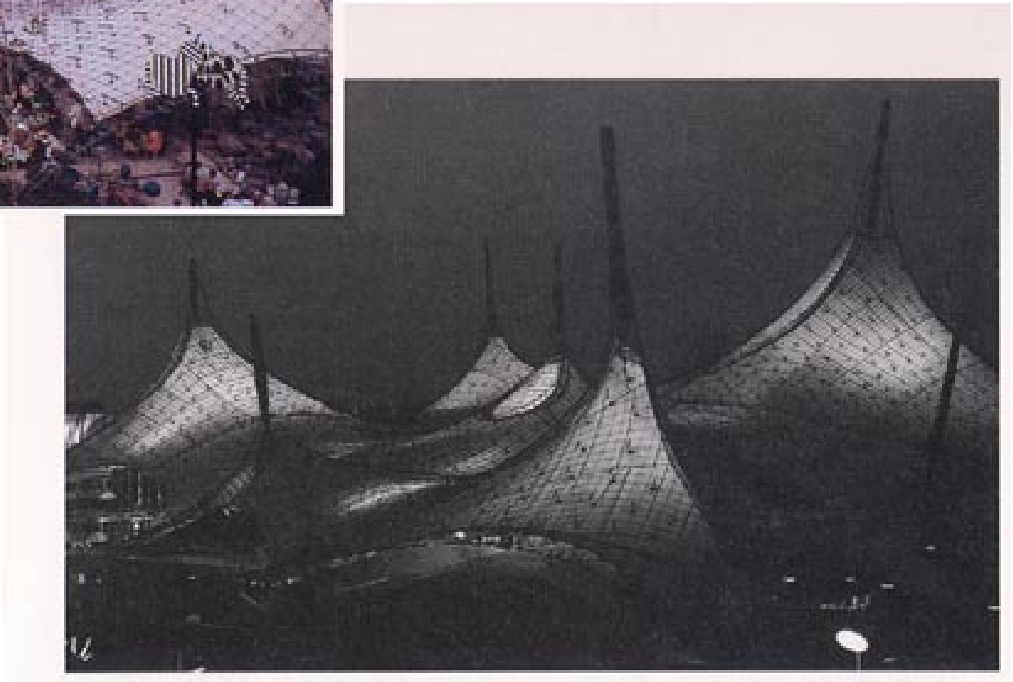


Kenzo Tange, Yoshikatsu Tsuboi, and Mamoru Kawaguchi

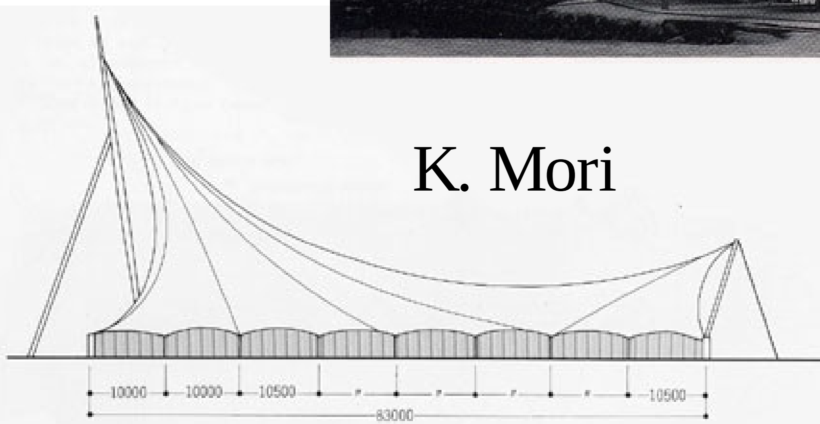
German Pavilion, Montreal EXPO 1967

Frei Otto, Rolf Gutbrod

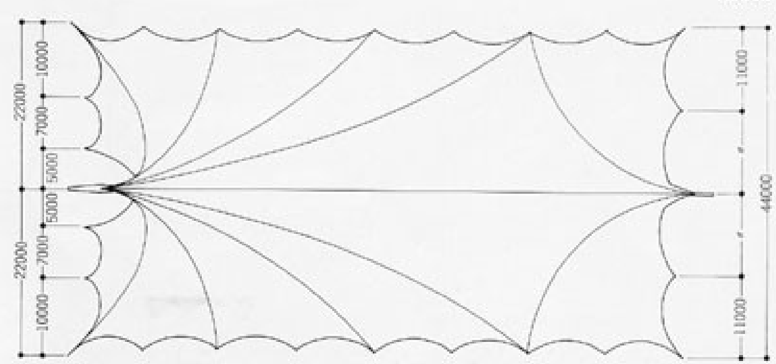




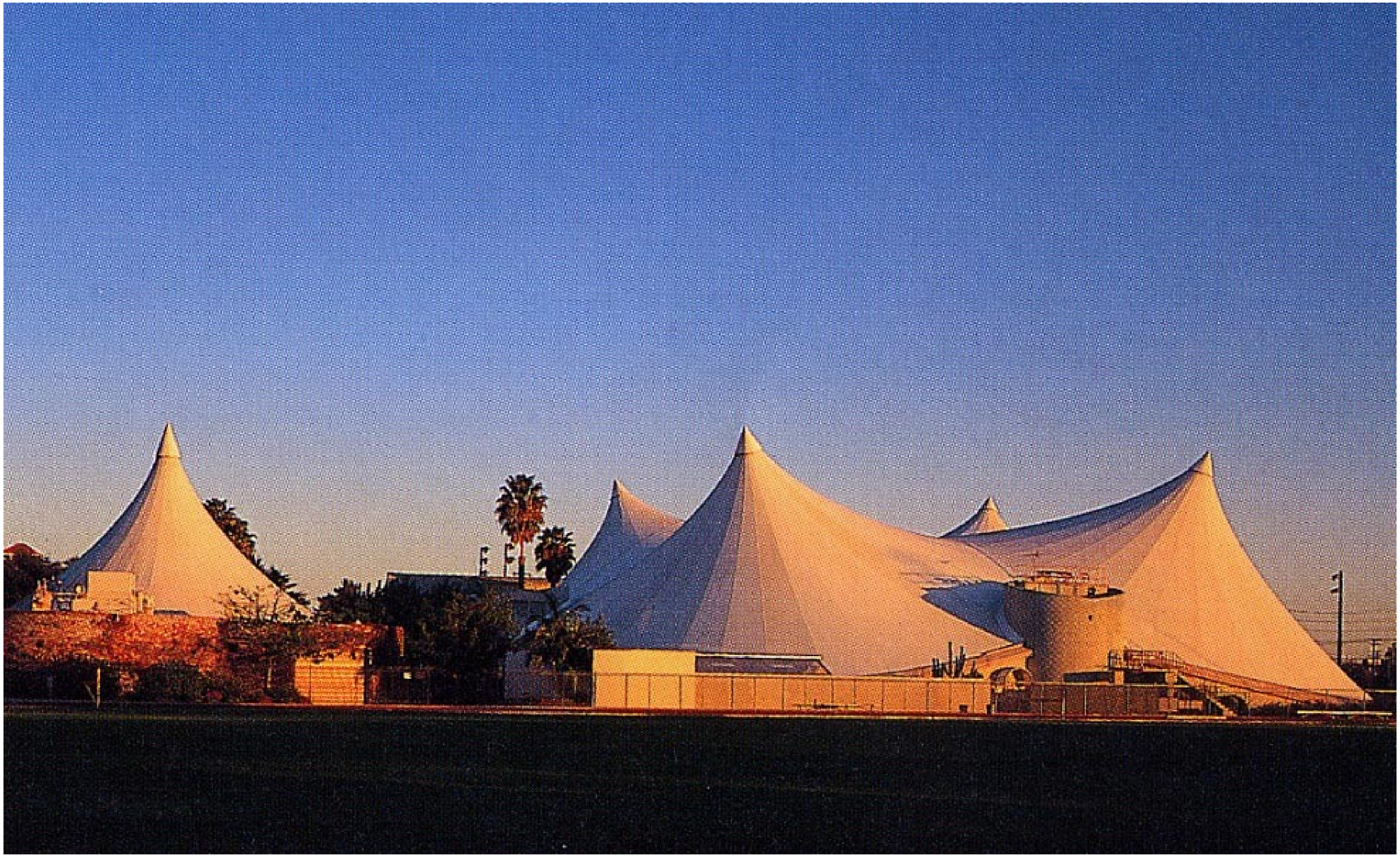
Young Land, Japan 1968



K. Mori



Student Center, La Verne (CA) 1973



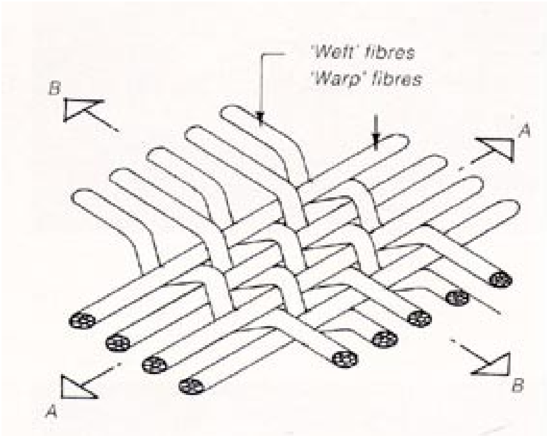
One of the first architectural applications of PTFE coated Fibreglass fabrics developed in 1972.
Fabric was tensile tested after 20 years at 70% fill/80% warp of original strength.

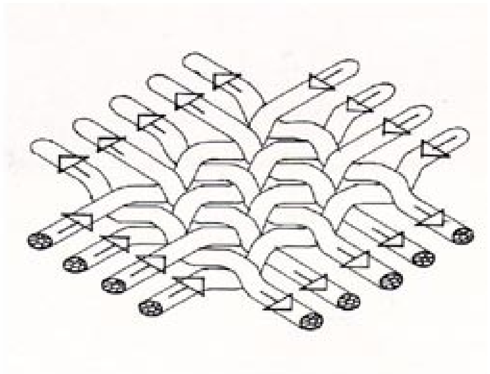
Haj Terminal, Saudi Arabia 1981

Horst Berger/Skidmore Owings & Merrill

Membrane Properties

Tensile only: no shear or compression





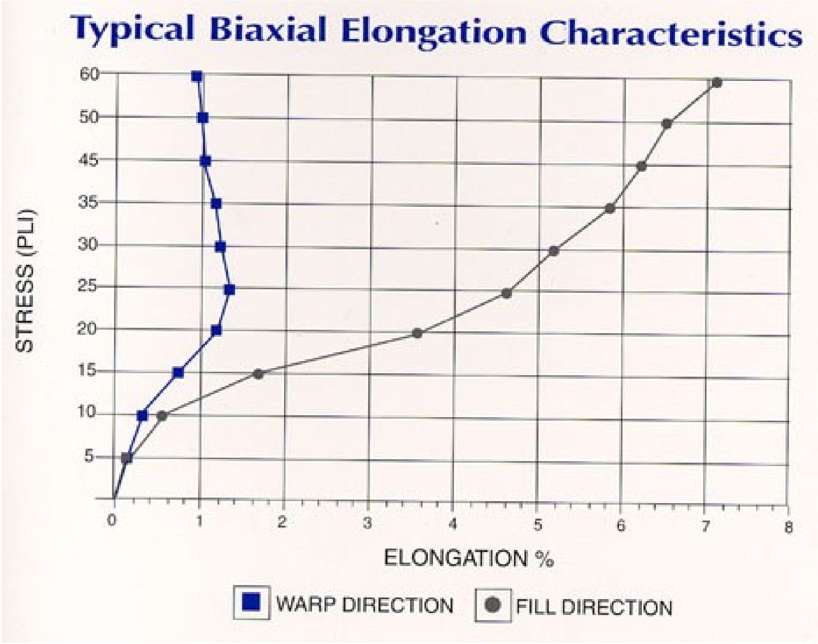
- <u>Strength</u>

(38.5 ounce per square yard PTFE coated Fibreglass Fabric)

Warp: 785 lb/in.

Fill: 560 lb/in.

- <u>Creep</u>



- <u>Modulus of Elasticity (E)</u>

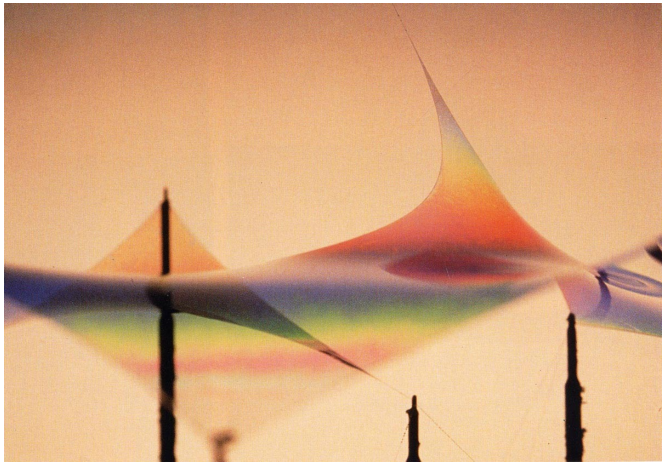
E=stress/strain

(stress=force/area,strain=dL/L)

- <u>Poisson's Ratio</u>: ratio of strain in x and y directions

Bi-axial testing of every roll of raw goods.

Equilibrium Conditions

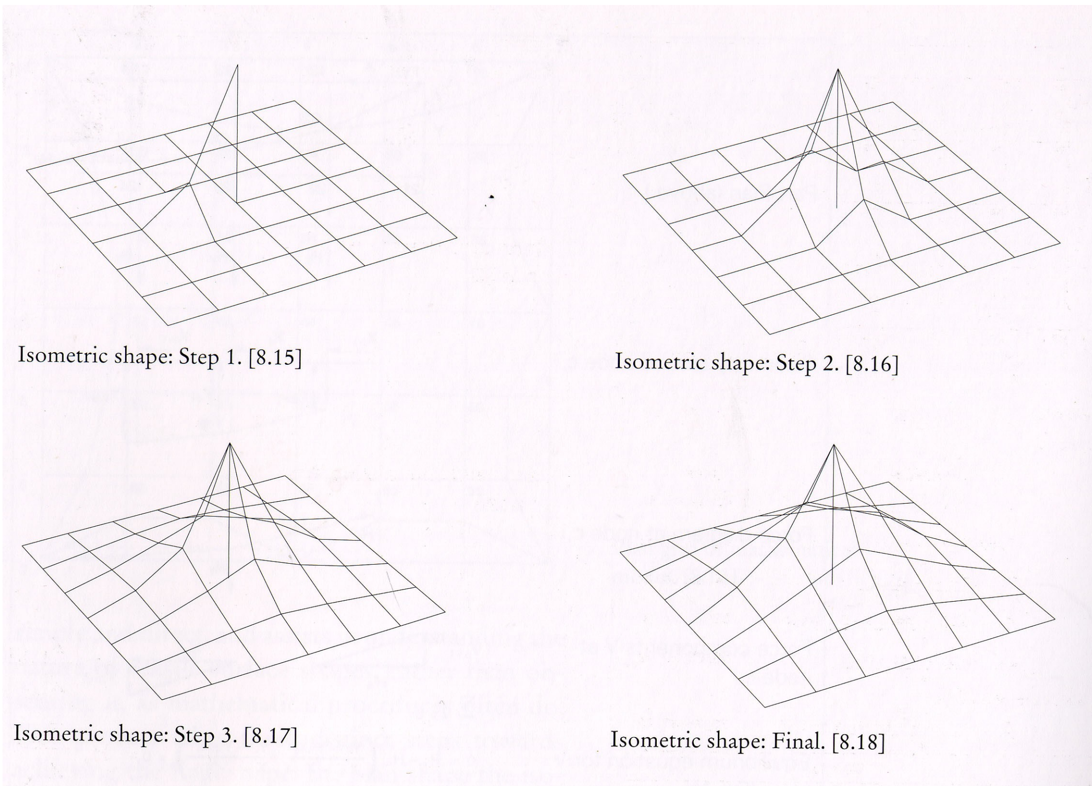


Soap bubbles are minimum surfaces with uniform surface tension. Early form finding work used 3D stereo-photography of soap bubbles and moiree methods.

Membrane Structures are optimized for structural live loads and are designed with a specified prestress, which affects the equilibrium shape. Support conditions, membrane stiffness, and biaxial properties are also factors in the final form.

Horst Berger's Grid Method

Used to create initial geometric form based on a equilibrium of forces by calculating values of the z coordinate using force balancing equilibrium equations.

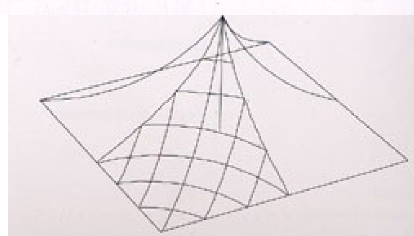


Isometric shape: Step 1. [8.15]

Isometric shape: Step 2. [8.16]

Isometric shape: Step 3. [8.17]

Isometric shape: Final. [8.18]



Final geodesic form (with added ridge cables)

1. Start with a plan view with a grid of nodes.

2. Enter starting elevation of center node.

3. Compute for equilibium of forces in sucessive surrounding nodes.

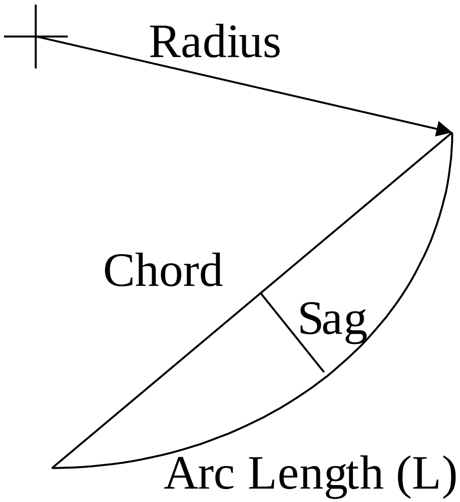
4. Reiterate new z coordinates into equilibrium equations.

5. Convert the isometric shape to a geodesic shape by rotating the coordinate system to be orthagonal at each node.

5. Reiterate new x, y, z coordinates into the equilibrium equations.

Simple Preliminary Analysis

2-Dimensional Example



Geometrically: $R=(C^2+4S)/8S$ and $C=2R\sin(L/2R)$

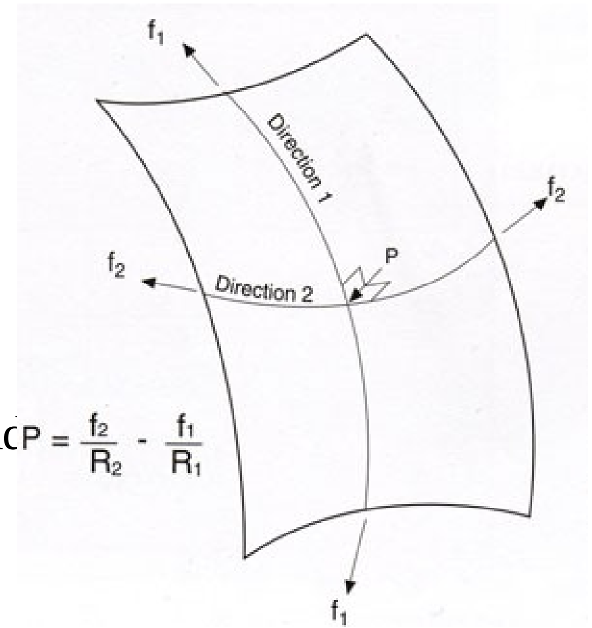
1. Calculate membrane tension for given pressure ($T=P*R$)

2.As membrane tension increases, membrane will stretch. $dL=T*L/wE$ (w=strip width). New length=L+dL

3. Iterate to find new radius based on new arc length.

4. Calculate new tension based on new new values.

- Moving to three dimensions requires solving equations simultaneously.
- Stress/strain is assumed to be linear.
- Equations are geometrically non-linear.
- Form finding is the process of determining the equilibrium shape with a given pre-stress, applied loads, and membrane properties.
- Most form finding programs are based on a 3-node triangular gric which is advantageous for subsequent patterning (though John Hollyday and David Salmon researched multi-noded elements at Cornell from 1983-1987).



Pioneers of Computerized Form Finding

1965: Alistair Day introduces Dynamic Relaxation method for analysis, later refined by Micheal Barnes, J Bunce, John Argyris, and David Wakefield ("AS Day, An Introductions to Dynamic Relaxation" The Engineer, V219 1965.)

1969: Early work by Ove Arup on the analysis of hanging roofs. (AS Day, and J Bunce).

1970: David Geiger associates and M. McCormick: first computer analysis of a fabric membrane of the air supported roof at US Pavilion at Expo in Osaka.

1969-71: Development of computer for form-finding for structures by Klaus Linkwitz (from work begun in 1966) calling it "The Stuttgart Direct Approach". Programmed on a CDC 6600 to design and analyze the Olympic Roofs in Munich.(Linkwitz and HJ Schek, "A New Method of Analysis of Prestressed Cable Networks", IABSE, Amsterdam, 1972.

1971: First published form-finding method of membranes with specified prestress: Micheal Barnes "Pretensioned Cable Networks, Construction Research and Development Journal, Vol. 3, No. 1, 1971

1973: Interactive form finding program on an IBM Mainframe by Massimo Majowiecki at STM from work deriving from 1970 thesis. Used to design the Coverture of the Rome stadium, Torin Stadium, and Athen Stadium.

1974: HJ Schek introduces Force Density Method, now used by Geiger,and in commercialy available programs Forten and Cadisi (Schek, " Force Density Methods for Form Finding and Computation of General Networks, Computer Methods in Applied Mechanics and Engineering, 1974)

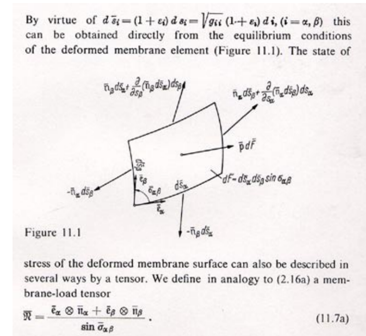
1975: Ross Dalland: Cornell thesis on form finding and patterning.

1980: Robert Haber: Cornell thesis, Stiffness method kernel for Birdair Images program.

1980: Buro Happold Tensyl Program

1981:First Computer Patterning (?) by Birdair (Minneapolis Metrodome), 1981.(Source: Geiger).

Early 1980's: William Spillers pioneers advanced stiffness method with material with non-linear properties used for the larger Berger/Geiger structures.



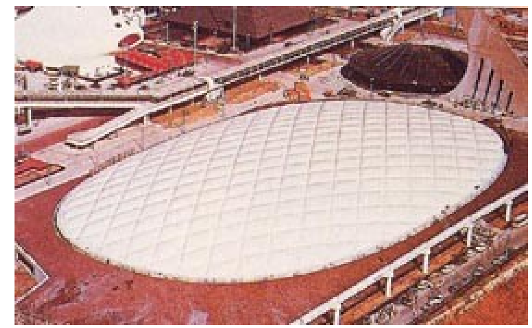
By virtue of $d\bar{s}_i = (1+\varepsilon_i)\,ds_i = \sqrt{g_{ii}}\,(1+\varepsilon_i)\,di,\ (i=\alpha,\beta)$ this can be obtained directly from the equilibrium conditions of the deformed membrane element (Figure 11.1). The state of

Figure 11.1

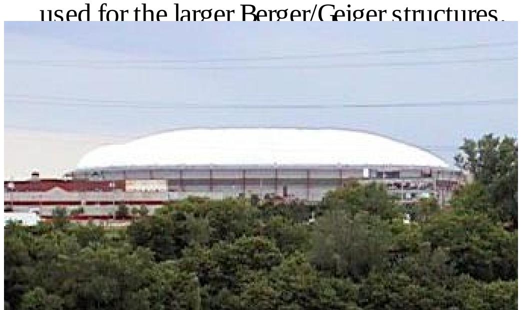
stress of the deformed membrane surface can also be described in several ways by a tensor. We define in analogy to (2.16a) a membrane-load tensor

$$\bar{\mathfrak{N}} = \frac{\bar{e}_\alpha \otimes \bar{n}_\alpha + \bar{e}_\beta \otimes \bar{n}_\beta}{\sin \bar{\sigma}_{\alpha\beta}}. \qquad (11.7a)$$

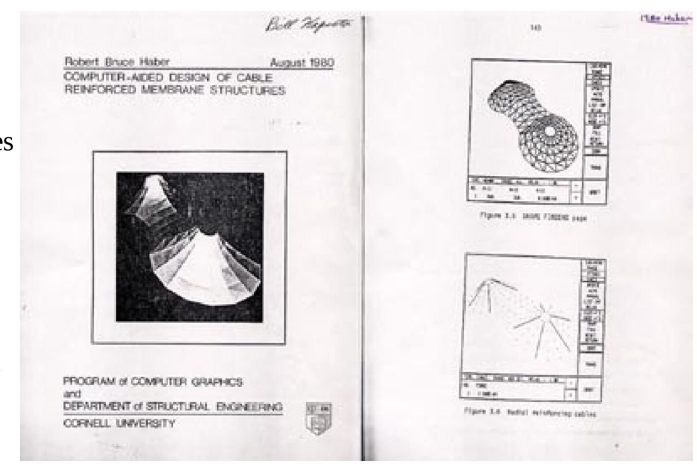
Frei Otto's MIT Thesis, 1962



Expo at Osaka, 1970



Left: Minneapolis Metrodome, 1981



Robert Bruce Haber August 1980

COMPUTER-AIDED DESIGN OF CABLE REINFORCED MEMBRANE STRUCTURES

PROGRAM of COMPUTER GRAPHICS and DEPARTMENT of STRUCTURAL ENGINEERING

CORNELL UNIVERSITY

Right: Robert Haber's Cornell thesis: Form Finding with graphical results.

Form Finding Methodologies

There are three main methods used to find the equilibrium shape. All lead to the same result, which is an minimum surface for a given pre-stress, membrane characteristics, and edge and support conditions. Modern programs can take into account structural characteristics of supports, uneven loading, and non-linear membrane characteristics.

For a constant membrane thickness taking into account the weight of the membrane, no curved surface exists whereby all points on the surface have equal tension. It is possible, however, to obtain a curved surface where the shearing force at every point is zero.

An important component of design is the analysis of the equilibrium surface, based on varying load scenarios. The final form the designer chooses may vary from the equilibrium surface so as to be optimized for estimated load extremes and considerations of on-site construction and pre-stressing methods.

1. Non-Linear Stiffness Matrix Analysis

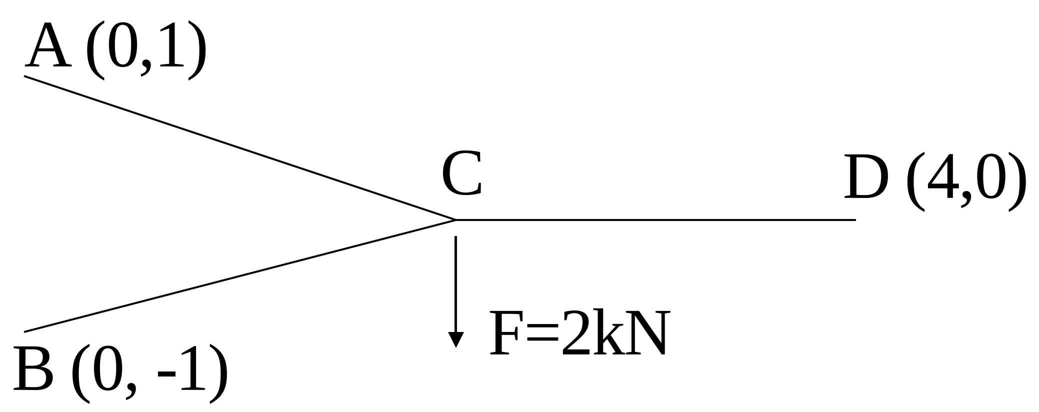
Based on matrix analysis of P=KU

- P=vector resultant prestress and applied nodal loads
- K=stiffness matrix (function of the directional modulus of elasticity)
- U=vector of nodal displacements.

External loads are given and equations are solved at each node simultaneously. Reiteration takes place until all the node forces are equal to the desired prestress. Stiffness matrix increases exponentially with the number of nodes.

2D example with rods.

Find: C(x,y) based on a target prestress F(i)



Solution:

Given: $A=40mm^2$ for each bar

$E=2*10^8$ kPa (uniform)

F(i)=10kN prestress for each bar (note: initial conditions are not in equilibrium)

1. Find resultant forces (P) around C (2 by 2 matrix).
2. Find stiffness matrix based on A, E (directional) and L (4 x 4 matrix)
3. Solve for U (2x2 matrix)
4. Find dL and change in force (=(dL*A*E)/L) for each member. Find new values for C(x,y). The equilibrium forces are not yet equal.
5. Scale deflections and iterate until all force vectors (Fca,Fcb, Fcd) approach the desired prestress.

2. Dynamic Relaxation

Dynamic Relaxation methods solve the geometric non-linear problem by equating it to a dynamic problem. Mass and damping characteristics are approximated. The prestress is fixed. Residual (non-equilibrium) forces result in a dynamic behavior at each node. Developed prior to high powered computers. Highly tolerant of poor initial form.

Dynamic Relaxation Method:

Residual force =sum of internal forces - applied load at each node.

$R(i)= Md^2y/dt^2+Ddy/dt$ (use Taylor series to approximate).

(D=coefficient of viscous damping)

To solve for equilibrium:

1. Solve for member forces using $F(i)=(dL*A*E)/L$ at each node.

2. Solve for residual forces geometrically and find velocity based on dynamic behavior.

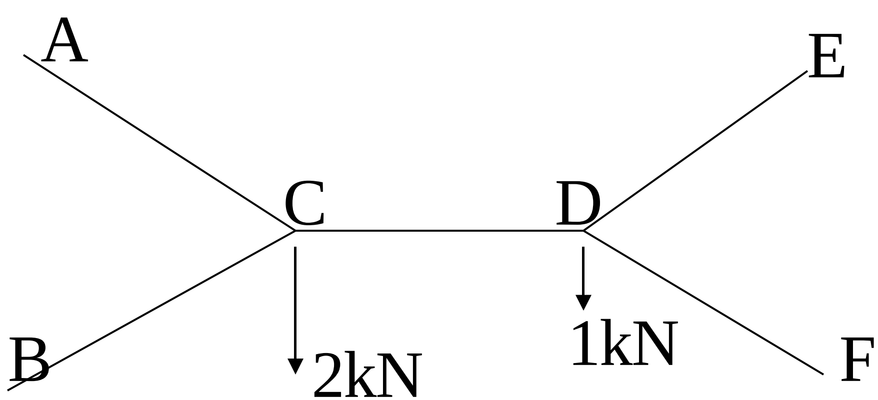
3. Find new position based on time increment (distance=velocity*time)

4. Reiterate until residual forces approach zero.

3. Force Density Methods

Force Density is a term given to the ratio of forces to lengths. The higher the force density ratio, the shorter the element for a given force. When the force densities for a node are equal and evenly distributed around the node, a minimal surface is generated. Once the equilibrium shape is determined, the stress-strain relationships are used to calculate the unstressed lengths. Non-linear equations are transformed into equivalent linear equations.

Example: Find C(x,y) and D (x,y)

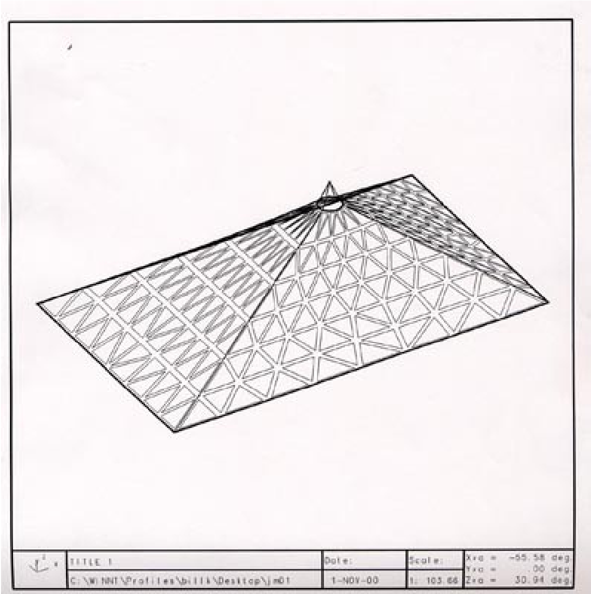


Solution:

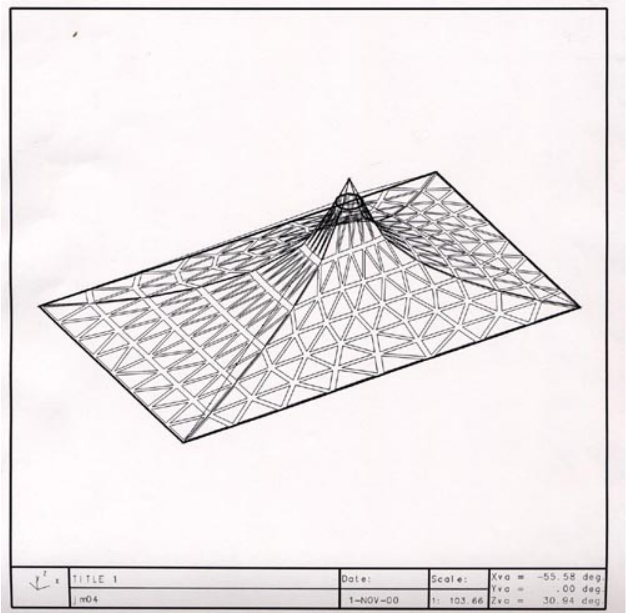
1. Sum forces around nodes C,D (4 equations).
2. Define Force Density g(i)=F(i)/L(i)
3. Use Linear Algebra to simlutaneously solve for C(x), D(x), C(y), D(y)
4. Solve for new lengths, and then for forces.
5. The unstressed length, used for patterning, can be calculated using the stressed length, the stress, and the stress/strain relationship:

L(original)=L(i) - (1.0 - F(i)/(A(i)E(i))

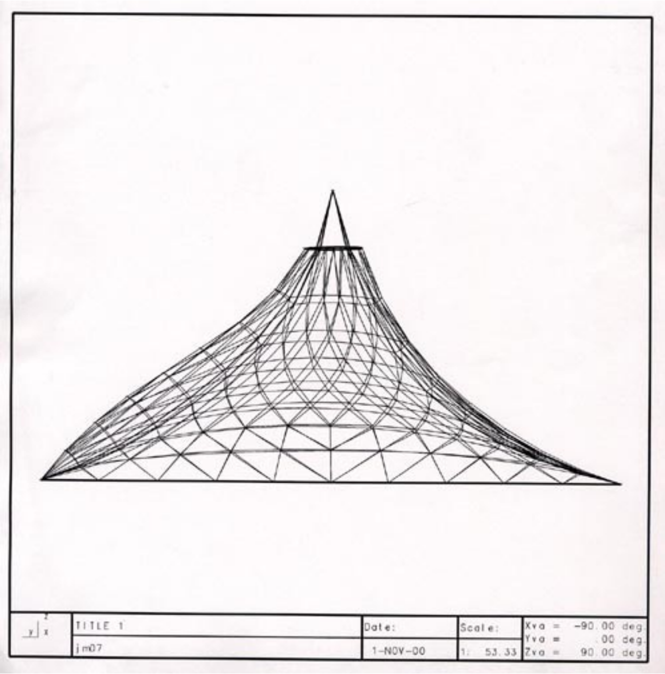
Birdair Images Program



Basic Parameters

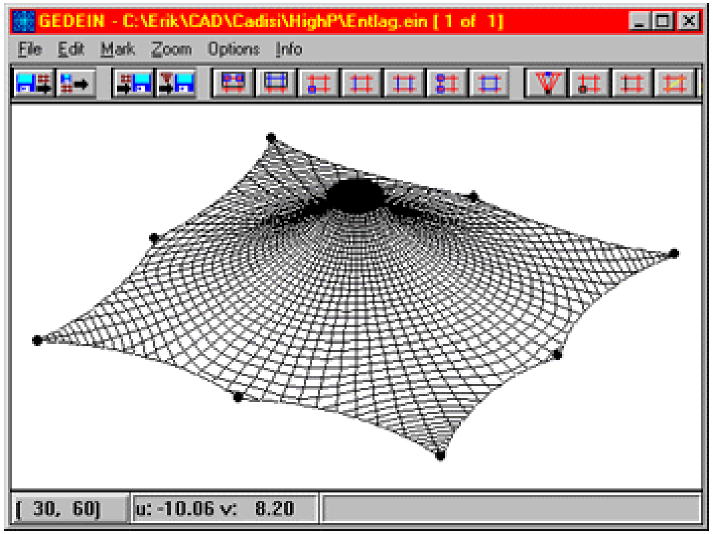


Form Finding

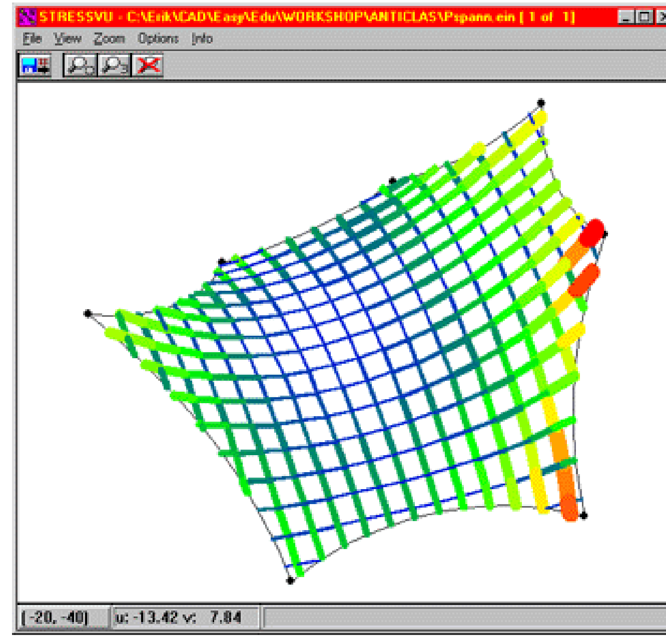


Load Analysis

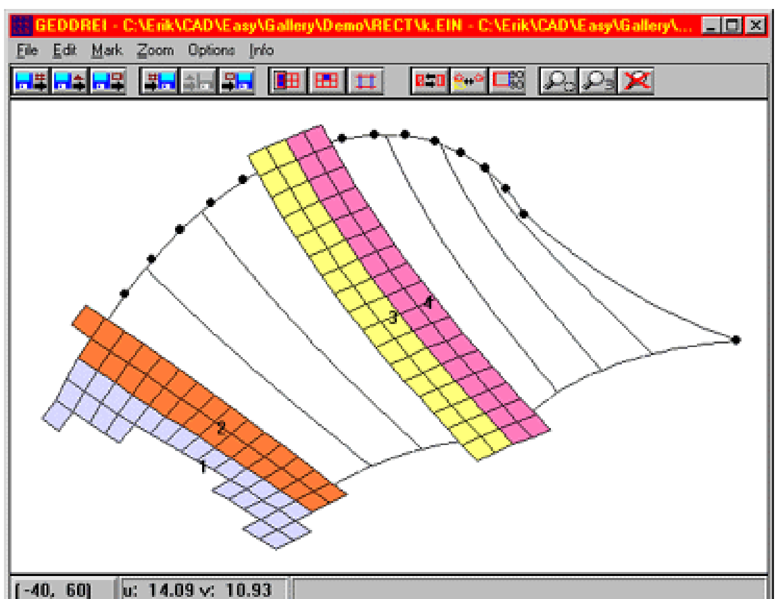
Modern Computer Programs



Form Finding

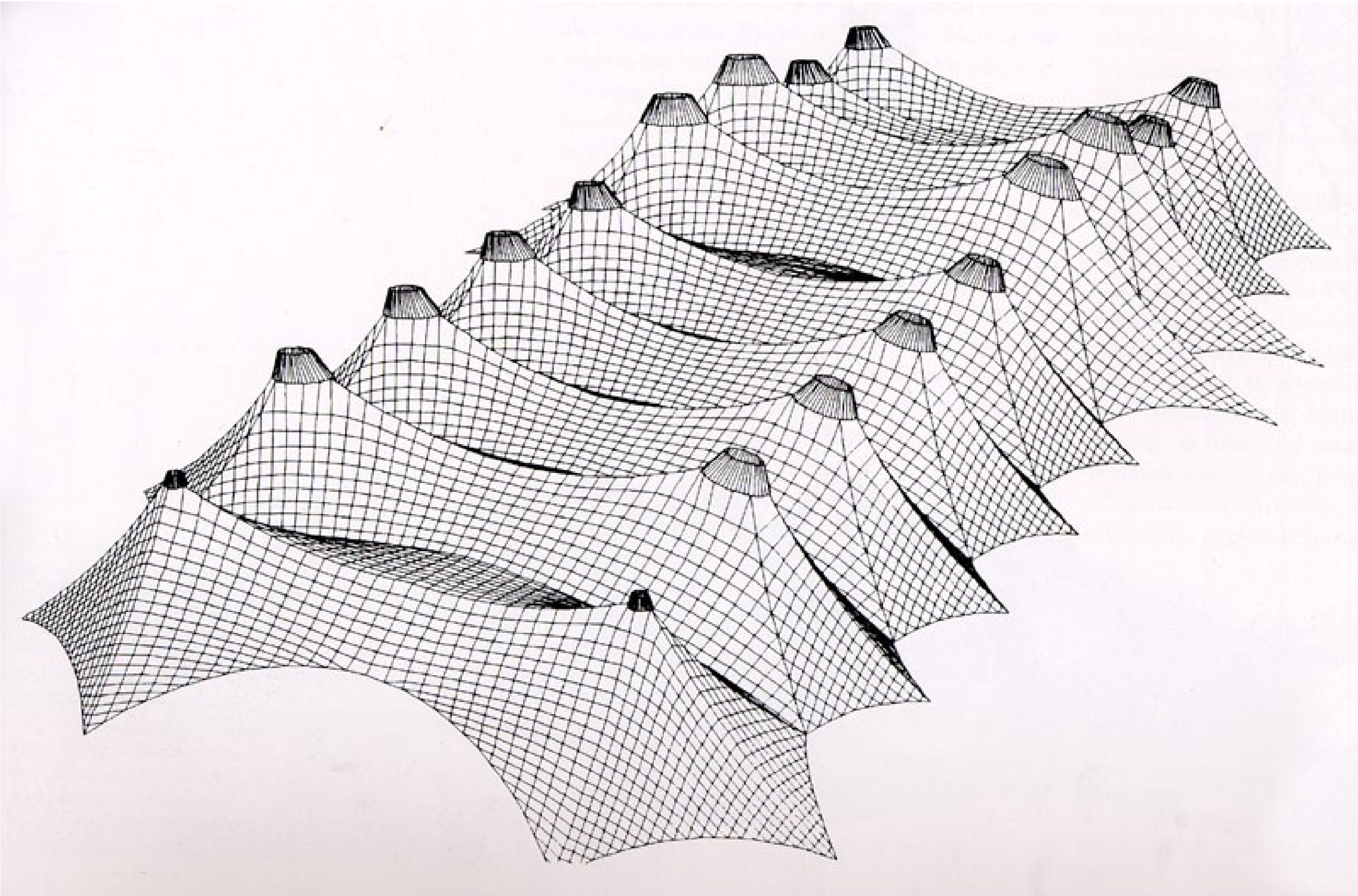


Stress Analysis



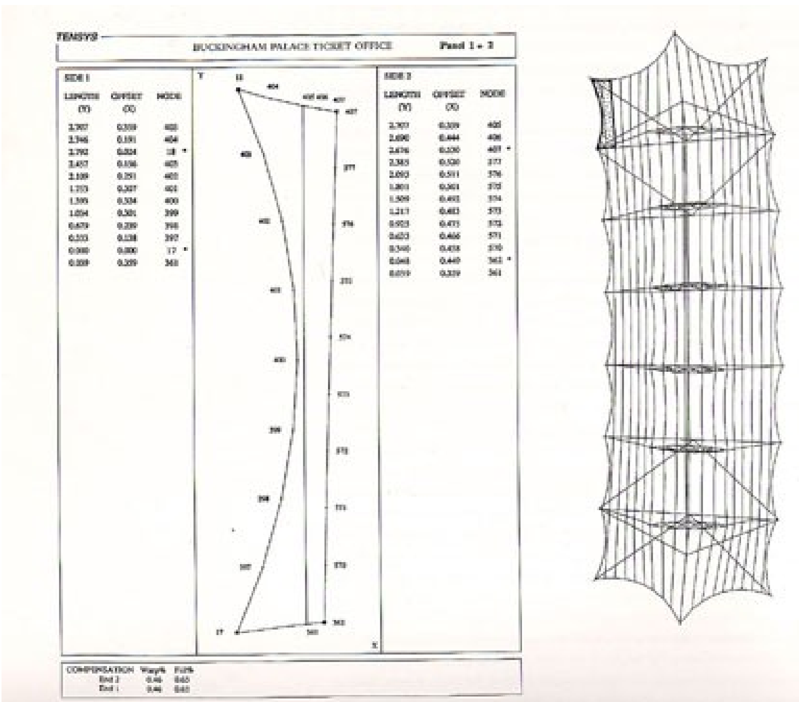
Patterning

Denver Airport

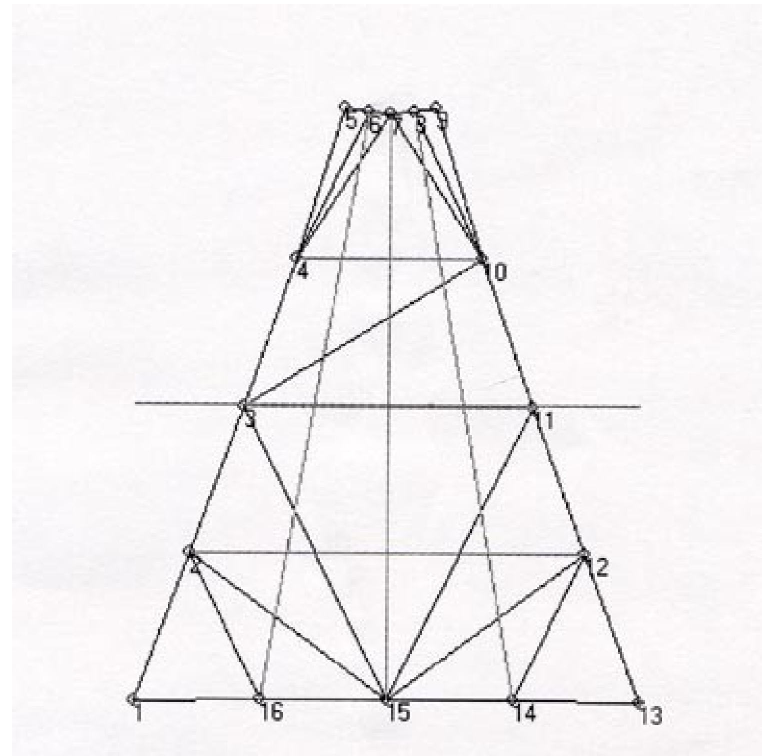


Analysis Program developed by William Spillers, NJIT

Patterning



Tensys (David Wakefield)



Pattern 2000 (Birdair) calculates lengths of geodesic curves and flattens triangles.

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•**The Structural Basis of Architecture**, Bjorn Sandaker and Arne Eggen, Whitney Library, 1992

•**The Science of Soap Films and Soap Bubbles**, Cyril Isenberg, Dover 1992

•**The Unique Role of Computing in the Design and Construction of Tensile Membrane Structures**, American Society of Civil Engineers, New York, 1991

Web Sites

Fabric Structure Building Companies
Birdair: http://www.birdair.com
Skidmore, Owings, Merrill: http://www.som.com
Ove Arup: http://www.arup.com/
Buro Happold: http://www.burohappold.com/
Geiger Engineers: http://www.geigerengineers.com/
Schlaich Bergermann: http://www.sbp.de/
Tentnology: http://www.tentnology.com
Fabric Manufacturers
Seaman: http://www.architecturalfabrics.com/whitepaper.html
Chemfab: http://www.chemfab.com/chemglas.htm
Software
Technet GmbH: http://www.technet-gmbh.com/
Ceafab; http://www2.tpg.com.au/users/simi/ceafab.htm
Forten: http://www.forten32.com/
Surface: http://www.surface.co.nz
Organizations
American Society of Civil Engineers: http://www.asce.org/
International Fabrics Association: http://www.ifai.com/
General
NJIT Introduction to Fabric Structures: http://www-ec.njit.edu/civil/gateway.html
Curvilinear Surfaces: http://www.curvedsurfaces.com/
Great Buildings: http://www.greatbuildings.com/
International Database of Structures: http://www.structurae.de/
Tensile Structures Yellow Pages: http://members.tripod.com/forten32/tsyp.html

Additional References

Contacts:

Horst Berger (City College of New York)

William Spillers (New Jersey Institute of Technology)

Gerry D'Anza (FORTEN)

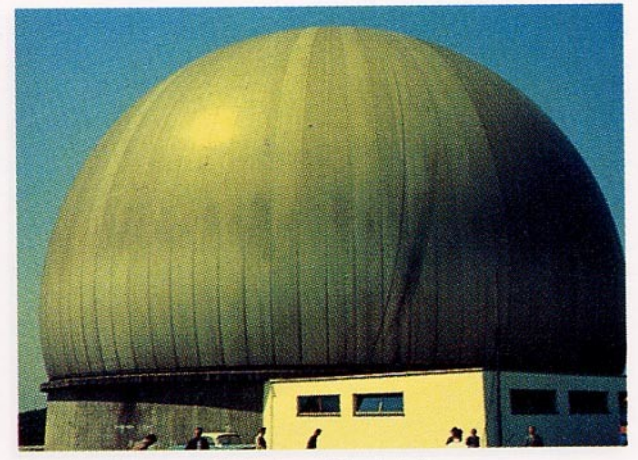
Angus Palmer (Happold)

William Kaputa (Birdair)

Slade Gellen (Birdair)

Michael Barnes (University of Bath)

Klaus Linkwitz (University of Stuttgart)



Radar Dome. (Photo: Birdair)